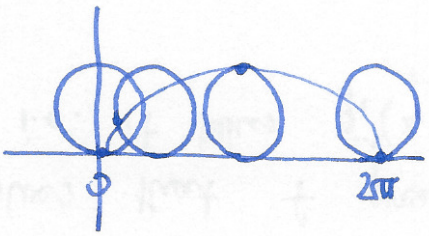
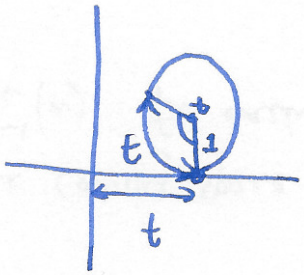
Example cycloid





$x(t) = t - \sin t \qquad \frac{dx}{dt} = 1 - \cos t$

$y(t) = 1 - \cos t \qquad \frac{dy}{dt} = \sin t$

arc length: $\int_0^{2\pi} \sqrt{(1-\cos t)^2 + \sin^2 t}\, dt = \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}\, dt$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$

recall: $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

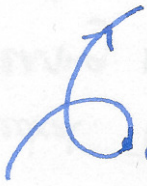
$2\sin^2\theta = 1 - \cos 2\theta$

$2\sin^2\left(\frac{t}{2}\right) = 1 - \cos t$

$= \int_0^{2\pi} \sqrt{4\sin^2\frac{t}{2}}\, dt$

$= \int_0^{2\pi} 2\left|\sin\frac{t}{2}\right| dt = 4\int_0^{\pi} \sin\frac{t}{2}\, dt = 4\left[-2\cos\frac{t}{2}\right]_0^{\pi} = 8(0+1) = 8$

speed



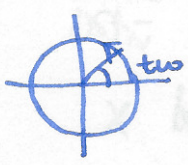
$c(t) = (x(t), y(t))$

$\text{speed} = \frac{d}{dt}(\text{arc length}) = \frac{d}{dt}\int_0^t \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

Example ① $c(t) = (t, t^2)$ $\quad x(t) = t$, $\frac{dx}{dt} = 1$; $\quad y(t) = t^2$, $\frac{dy}{dt} = 2t$

speed $= s'(t) = \sqrt{1 + 4t^2}$

Example ② $c(t) = (R\cos\omega t, R\sin\omega t)$



$c'(t) = (-\omega R\sin\omega t, \omega R\cos\omega t)$

$\text{speed} \quad \|c'(t)\| = \sqrt{R^2\sin^2\omega t + R^2\cos^2\omega t} = R\omega$