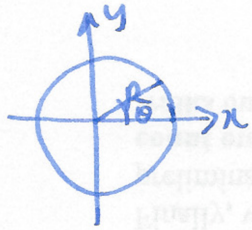
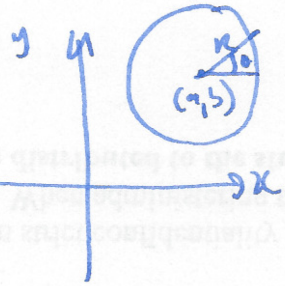
② circle of radius R, centered at (a,b).

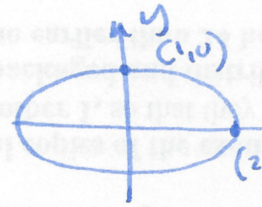


$$c(\theta) = (R\cos\theta, R\sin\theta)$$

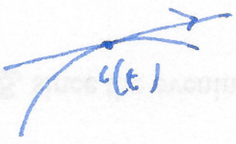


$$c(\theta) = (a + R\cos\theta, b + R\sin\theta)$$

③ ellipse



$$c(\theta) = (2\cos\theta, \sin\theta)$$



Q: how do we find the slope of the tangent line?

A: $\boxed{\dfrac{dy}{dx} = \dfrac{dy}{dt} \Big/ \dfrac{dx}{dt}}$ why: $c(t) = (x(t), y(t))$

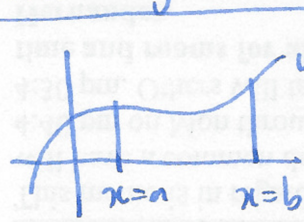
chain rule: $\dfrac{dy}{dt} = \dfrac{dy}{dx}\dfrac{dx}{dt} \Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{dt}\Big/\dfrac{dx}{dt}$

Example $c(t) = (\cos t, \sin 2t)$

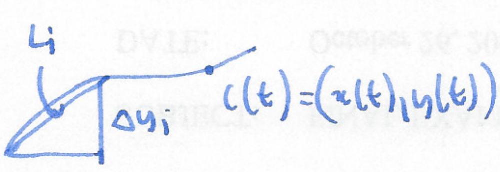
$$\left.\begin{array}{ll} x(t) = \cos t & y(t) = \sin 2t \\ \dfrac{dx}{dt}(t) = -\sin t & \dfrac{dy}{dt} = 2\cos 2t \end{array}\right\} \quad \frac{dy}{dx} = \frac{2\cos 2t}{-\sin t}$$

§ 11.2 Arc length and speed

recall



arc length $= \displaystyle\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$



$$\text{arc length} = \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$$

$$= \sum_{i=1}^{n} \sqrt{\left(\frac{\Delta x_i}{\Delta t_i}\right)^2 + \left(\frac{\Delta y_i}{\Delta t_i}\right)^2}\, \Delta t_i$$

take limit as $|\Delta t_i| \to 0$

$$\text{arc length} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$$