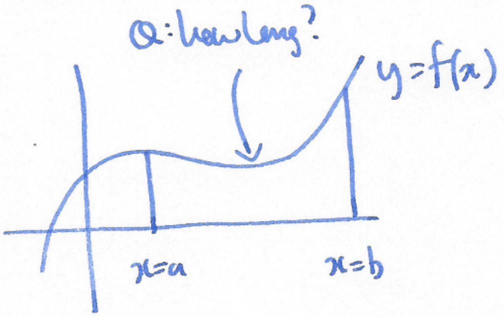
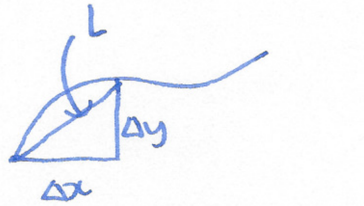
§8.1 Arc length and surface area

Q: how long?



A:

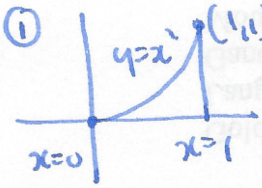


$$|L| = \sqrt{(\Delta x)^2+(\Delta y)^2}$$

$$\text{length} = \sum_{i=1}^{n} |L_i| = \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2+(\Delta y_i)^2} = \sum_{i=1}^{n} \sqrt{1+\left(\frac{\Delta y_i}{\Delta x_i}\right)^2}\,\Delta x_i$$

take limit as $|\Delta x_i| \to 0$: arc length $= \displaystyle\int_a^b \sqrt{1+(f'(x))^2}\,dx = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

Example ①



$\frac{dy}{dx} = 2x$ arc length $\displaystyle\int_0^1 \sqrt{1+4x^2}\,dx$

(trig. sub. integral)

② $f(x) = \frac{1}{12}x^3 + \frac{1}{x}$ on $[1,2]$

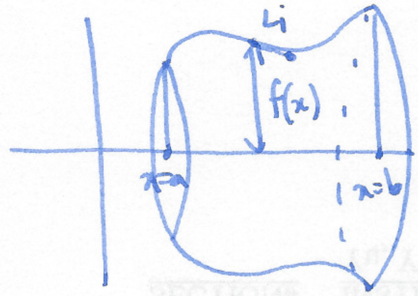
$f'(x) = \frac{1}{4}x^2 - \frac{1}{x^2}$

$$\int_1^2 \sqrt{1+\left(\frac{1}{4}x^2-\frac{1}{x^2}\right)^2}\,dx = \int_1^2 \sqrt{1+\frac{1}{16}x^4-\frac{1}{2}+\frac{1}{x^4}}\,dx$$

$$= \int_1^2 \sqrt{\frac{1}{16}x^4+\frac{1}{2}+\frac{1}{x^4}}\,dx = \int_1^2 \sqrt{\left(\frac{x^2}{4}+\frac{1}{x^2}\right)^2}\,dx = \int_1^2 \frac{x^2}{4}+\frac{1}{x^2}\,dx$$

$$\left[\frac{x^3}{12}-\frac{1}{x}\right]_1^2 = \frac{8}{12}-\frac{1}{4}-\frac{1}{12}+1 = 1\frac{1}{3}$$

Area of surface of revolution

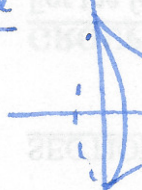


$$\text{area} \approx \sum |L_i| \, f(x) \, 2\pi.$$

take limit as $|\Delta x_i| \to 0$

$$\text{surface area of vol of revolution} = \int_a^b 2\pi f(x) \sqrt{1+(f'(x))^2}\, dx$$

Example



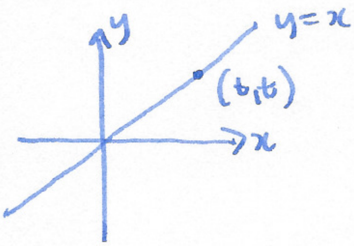
$y = e^{-x}$ $\quad \frac{dy}{dx} = -e^{-x}$ $\quad$ surface area $\int_0^\infty 2\pi e^{-x} \sqrt{1+(-e^{-x})^2}\, dx$

sub $u = e^{-x}$ etc....

$\frac{du}{dx} = -e^{-x}$

$$\int_0^\infty 2\pi e^{-x} \sqrt{1+u^2} \cdot \frac{1}{-e^{-x}}\, du = \int_0^1 2\pi \sqrt{1+u^2}\, du \quad \text{(trig sub. integral)}$$

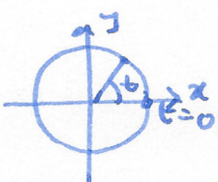
§11.1 Parametric equations



$x(t) = t$ $\qquad c(t) = (x(t), y(t))$

$y(t) = t$ $\qquad \uparrow$ parameterization.

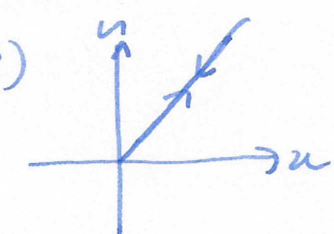
e.g.



$c(t) = (\cos t, \sin t)$

problem: many different parameterizations give the same curve.

e.g. $c(t) = (2t, 2t)$ gives $y=x$, as does $c(t) = (t^3, t^3)$



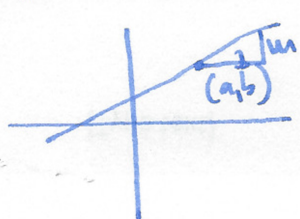
warning $c(t) = (t^2, t^2)$ not the same

Example describe the parametric curve $c(t) = (t+1, t^2+1)$ in terms of $y = f(x)$:

$x = t+1 \Rightarrow t = x-1$

$y = t^2+1 \Rightarrow y = (x-1)^2+1 = x^2-2x+2$

Example ① find a parametric equation for the line through (a,b) with slope m



$t=0: (a,b)$

$t=1: (a+1, b+m)$ $\qquad c(t) = (a+t, b+tm)$