Example $\int_{-\infty}^{\infty} \sin(x)\,dx$ DNE $\quad \lim_{R\to\infty}\int_{-R}^{R}\sin(x)\,dx = \lim_{R\to\infty}\left[-\cos(x)\right]_{-R}^{R}$

$= \lim_{R\to\infty} -\cos(R)+\cos(-R) = \lim_{R\to\infty} 0 = 0.$

Example $\int_0^\infty xe^{-x}\,dx = \lim_{R\to\infty}\int_0^R xe^{-x}\,dx$ $\qquad$ parts $\int uv'\,dx = uv - \int u'v\,dx$

$= \lim_{R\to\infty}\left[x\cdot(-e^{-x})\right]_0^R + \int_0^R e^{-x}\,dx$

$= \lim_{R\to\infty} -Re^{-R} + 0 + \left[-e^{-x}\right]_0^R = \lim_{R\to\infty} -Re^{-R} - e^{-R} + e^0 = 1$

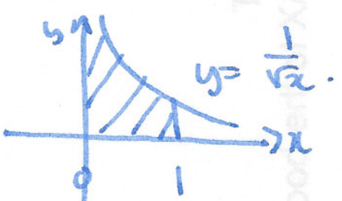
Example when does $\int_1^\infty \frac{1}{x^p}\,dx$ exist? $\left(\begin{array}{ll} p=1 & \text{no} \\ p=2 & \text{yes}\end{array}\right)$.

$\lim_{R\to\infty}\int_1^R \underbrace{\frac{1}{x^p}}_{x^{-p}}\,dx = \lim_{R\to\infty}\left[\frac{x^{-p+1}}{-p+1}\right]_1^R = \lim_{R\to\infty}\frac{R^{-p+1}}{-p+1} - \frac{1}{-p+1}$

$\lim_{R\to\infty} R^{-p+1} = 0$ if $p>1$

$= \infty$ if $p<1$

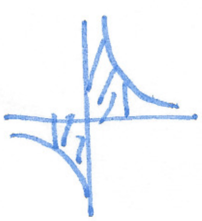
Integrals w/ discontinuities



$\int_0^1 \frac{1}{\sqrt{x}}\,dx$ is an improper integral! as $f(0)$ not defined

$= \lim_{R\to 0}\int_R^1 \underbrace{\frac{1}{\sqrt{x}}}_{x^{-1/2}}\,dx = \lim_{R\to 0}\left[2x^{1/2}\right]_R^1 = \lim_{R\to 0} 2-2\sqrt{R} = 2$

Warning: $\underbrace{\int_{-1}^{1}\frac{1}{x}\,dx}_{\text{DNE!}} \neq \left[\ln|x|\right]_{-1}^{1} = \ln|1| - \ln|-1| = 0.$ wrong!.



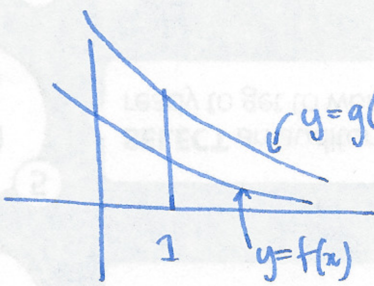
interval $[-1,1]$ contains discontinuity for $\frac{1}{x}$.

$$\int_{-1}^{1} \frac{1}{x}\,dx = \int_{-1}^{0} \frac{1}{x}\,dx + \int_{0}^{1} \frac{1}{x}\,dx = \lim_{R\to 0} \int_{-1}^{R} \frac{1}{x}\,dx + \lim_{R\to 0} \int_{0}^{1} \frac{1}{x}\,dx$$

$$= \lim_{R\to 0} \Big[\ln|x|\Big]_{-1}^{R} + \lim_{R\to 0} \Big[\ln|x|\Big]_{R}^{1}$$

$$= \lim_{R\to 0} \underbrace{\ln|R| - 0}_{\to\infty} + \lim_{R\to 0} \underbrace{\ln 0 - \ln|R|}_{\to -\infty} \quad \infty - \infty. \ \text{DNE}.$$

<u>Comparison test</u>



suppose $0 \le f(x) \le g(x)$ on $[1,\infty)$

if $\int_1^\infty g(x)\,dx$ converges, then $\int_1^\infty f(x)\,dx$ converges.

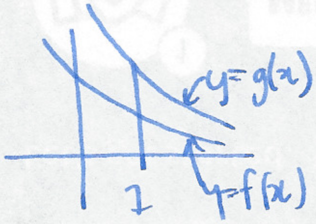
<u>Example</u> $\int_1^\infty \frac{1}{\sqrt{x^3+1}}\,dx$ note: $\sqrt{x^3+1} \ge \sqrt{x^3} \ge 0$ on $[1,\infty)$.

so $0 \le \frac{1}{\sqrt{x^3+1}} \le \frac{1}{\sqrt{x^3}}$

<u>try</u> $\int_1^\infty \frac{1}{\sqrt{x^3}}\,dx = \int_1^\infty x^{-3/2}\,dx = \lim_{R\to\infty} \Big[-2x^{-1/2}\Big]_1^R = \lim_{R\to\infty} -\frac{2}{\sqrt{R}} + 2 = 2 < \infty$ converges

$\Rightarrow \int_1^\infty \frac{1}{\sqrt{x^3+1}}\,dx$ converges. (but don't know exact value)

<u>other way</u>



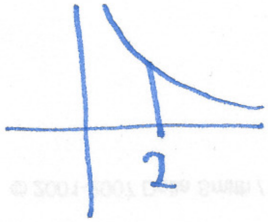
$0 \le f(x) \le g(x)$ on $[1,\infty)$

if $\int_1^\infty f(x)\,dx$ diverges $\Rightarrow \int_1^\infty g(x)\,dx$ diverges

<u>note</u>: $\int_1^\infty g(x)\,dx$ diverges $\not\Rightarrow \int_1^\infty f(x)\,dx$ diverges

$\int_1^\infty f(x)\,dx$ converges $\not\Rightarrow \int_1^\infty g(x)\,dx$ converges.

Example



show $\displaystyle\int_2^\infty \frac{1}{x-\sqrt{x}}\,dx$ diverges

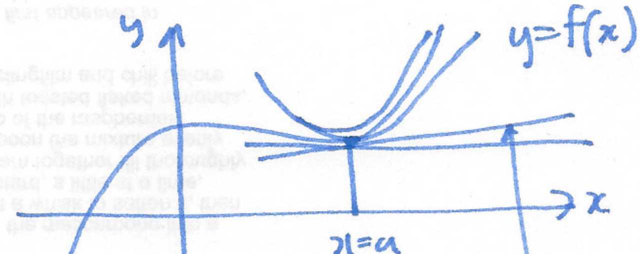
$$x-\sqrt{x} < x \quad (\text{for } x>1)$$

$$\frac{1}{x-\sqrt{x}} > \frac{1}{x}$$

$$\int_2^\infty \frac{1}{x}\,dx = \lim_{R\to\infty}\int^R \frac{1}{x}\,dx = \lim_{R\to\infty}\Big[\ln|x|\Big]_1^R = \lim_{R\to\infty} \ln|R| - 0 \to \infty$$

$$\Rightarrow \int_2^\infty \frac{1}{x-\sqrt{x}}\,dx \text{ diverges.}$$

§8.4 Taylor Polynomials



Approximating functions at $x=a$:

tangent line
$y-y_0 = m(x-x_0)$
$y-f(a) = f'(a)(x-a)$

0-th approx: $f(x) \simeq f(a)$

1-st approx: (straight line) $f(x) \simeq f(a) + f'(a)(x-a)$

2-nd approx: (quadratic) ? how do we find these ?

3-rd approx: (cubic)

- choose quadratic where first 2 derivatives at a agree with f.
- choose cubic " 3 " "
- etc.

<u>quadratic</u> $T_2(x) = a_0 + a_1(x-a) + a_2(x-a)^2$

deg 0: $T_2(a) = a_0 \qquad = f(a)$

deg 1: $T_2'(x) = a_1 + 2a_2(x-a)$

$T_2'(a_0) = a_1 \qquad = f'(a)$