<u>Example</u> $\int \sin^4 x \cos^3 x \, dx$ try: $u = \sin x$

$\frac{du}{dx} = \cos x$

$= \int \sin^4 x \cos^2 x . \cos x \, dx = \int \sin^4 x (1 - \sin^2 x) . \cos x \, dx$

$= \int u^4 (1 - u^2) \, du = \int u^4 - u^6 \, du = \frac{1}{5} u^5 - \frac{1}{7} u^7 + c$

<u>even powers</u> $\int \sin^4 x \cos^2 x \, dx$ get everything in terms of sin(x) or cos(x), then use parts

$\int \sin^4 x (1 - \sin^2 x) \, dx = \int \sin^4 x - \sin^6 x \, dx$

$\int \sin^6 x \, dx = \int \underset{u}{\sin^5 dx} \, \underset{v'}{\sin x} \, dx$ $u = \sin^5 x$ $u' = 5 \sin^4 x \cos x$

$v' = \sin x$ $v = -\cos x$

$= uv - \int u'v \, dx$

$= -\sin^5 x \cos x + \int 5 \sin^4 x \cos^2 x \, dx$

$= -\sin^5 x \cos x + 5 \int \sin^4 x (1 - \sin^2 x) \, dx$

$\int \sin^6 x \, dx = -\sin^5 x \cos x + 5 \int \sin^4 x \, dx - 5 \int \sin^6 x \, dx$

$6 \int \sin^6 x \, dx = -\sin^5 x \cos x + 5 \underbrace{\int \sin^4 x \, dx}_{\text{do by parts!}}$

## other trig functions

recall: $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx \quad \begin{array}{l} u = \cos x \\ \frac{du}{dx} = -\sin x \end{array} = \int \frac{\sin x}{u} \cdot \frac{-1}{\sin x} du$

$= -\int \frac{1}{u} du = -\ln|u| + c = -\ln|\cos x| + c = \ln|\sec x| + c$

fact: $\int \sec(x) dx = \ln|\sec x + \tan x| + c$

## other trig function powers

$\int \tan^a x \sec^b x \, dx$

use:
- $\cos^2 x + \sin^2 x = 1 \leftrightarrow 1 + \tan^2 x = \sec^2 x$
- $u = \sec x \quad \frac{du}{dx} = \sec x \tan x$
- $u = \tan x \quad \frac{du}{dx} = \sec^2 x$
- parts.

a odd: $\int \tan^3 x \sec^2 x \, dx = \int \tan^2 x \sec x \left( \tan x \sec x \right) dx$

$= \int (1 - \sec^2 x) \sec x \left( \tan x \sec x \right) dx \quad \begin{array}{l} u = \sec x \\ \frac{du}{dx} = \sec x \tan x \end{array}$

$= \int (1 - u^2) u \frac{\tan x \sec x}{\tan x \sec x} du = \int u - u^3 du = \frac{1}{2}u^2 - \frac{1}{4}u^4 + c$

$= \frac{1}{2}\sec^2 x - \frac{1}{4}\sec^4 x + c$

b even: $\int \tan^3 x \sec^2 x \, dx \quad \begin{array}{l} u = \tan x \\ \frac{du}{dx} = \sec^2 x \end{array}$

$= \int u^3 \frac{\sec^2 x}{\sec^2 x} du = \frac{1}{4}u^4 + c = \frac{1}{4}\tan^4 x + c$

$a$ even, $b$ odd : write as powers of $\sec(x)$ and use integration by parts. (18)

Example $\int \sin 3x \cos 2x \, dx$ ? useful fact $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$\left.\begin{aligned} \sin(A+B) &= \sin A \cos B + \cos A \sin B \\ \sin(A-B) &= \sin A \cos B - \cos A \sin B \end{aligned}\right\} \quad \sin(A+B) + \sin(A-B) = 2 \sin A \cos B$$

$$\int \sin 3x \cos 2x \, dx = \frac{1}{2} \int \sin(3x+2x) + \sin(3x-2x) \, dx$$

$$= \frac{1}{2} \int \sin 5x - \sin x \, dx = -\frac{1}{10} \cos 5x - \frac{1}{2} \cos x + C .$$

Example $\int \cos 4x \cos 7x \, dx$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$$

$$= \frac{1}{2} \int \cos(4x+7x) + \cos(4x-7x) \, dx = \frac{1}{2} \int \cos 11x + \cos 3x \, dx$$

$$= \frac{1}{2} \left( \frac{1}{11} \sin 11x + \frac{1}{3} \sin 3x \right) + C .$$

## §7.3 Trig substitutions

aim : deal with $\underset{①}{\sqrt{a^2-x^2}}$, $\underset{②}{\sqrt{a^2+x^2}}$, $\underset{③}{\sqrt{x^2-a^2}}$

$\cos^2 x + \sin^2 x = 1 \quad \leftrightarrow \quad \sin^2 x = 1 - \cos^2 x$ ①

$\leftrightarrow \quad 1 + \tan^2 x = \sec^2 x$ ②

$\leftrightarrow \quad \cot^2 x + 1 = \operatorname{cosec}^2 x \quad \leftrightarrow \quad \cot^2 x = \operatorname{cosec}^2 x - 1$ ③

Example $\int \sqrt{9-x^2} \, dx$ try $x = 3 \sin u$

$\frac{dx}{du} = 3 \cos u$