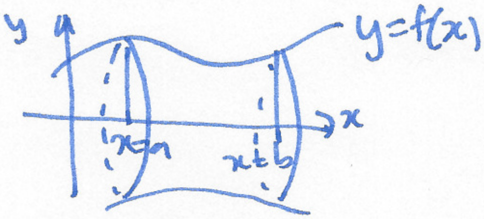
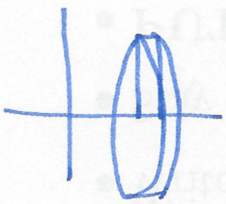
Surface area :



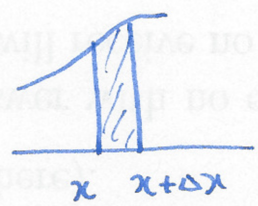
wrong:



area of cylinder $\approx 2\pi y \Delta x$

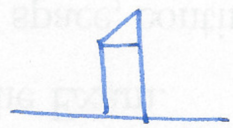
surface area $\approx \sum 2\pi y \Delta x \qquad 2\pi \int_a^b y\,dx$

why is this wrong?



area of strip $\approx y(x)\Delta x \qquad \frac{\text{error}}{\text{area}} \to 0$ as $\Delta x \to 0$

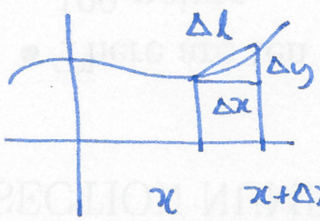
surface area like arc length:



$\frac{\text{error}}{\text{surface area}} \not\to 0$ as $\Delta x \to 0$

^ in fact constant for straight lines by similar triangles (arclength) (also surface area).

right way:



arc length $\sum \Delta l$

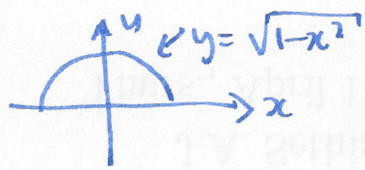
$(\Delta l)^2 = (\Delta x)^2 + (\Delta y)^2$

surface area $\approx y 2\pi \Delta l$

total surface area: $\sum 2\pi y \Delta l = \sum 2\pi\, y(x_i) \sqrt{(\Delta x_i)^2 + (\Delta y(x_i))^2}$ ← doesn't look like an integral!

$= 2\pi \sum y(x_i) \sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2}\,\Delta x \to$ surface area $= 2\pi \int_a^b y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

check: surface area of sphere:



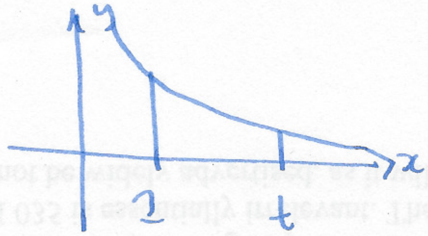
area $= 2\pi \int_{-1}^{1} \sqrt{1-x^2} \sqrt{1+\frac{dy}{dx^2}}\,dx$

$\frac{dy}{dx} = \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x)$

$2\pi \int_{-1}^{1} \sqrt{1-x^2} \sqrt{1+\frac{x^2}{1-x^2}}\,dx = 2\pi \int_{-1} \sqrt{1-x^2} \sqrt{\frac{1-x^2+x^2}{1-x^2}}\,dx$

$= \int_{-1}^{1} \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}\,dx = 2\pi \int_{1}^{1} dx = 2\pi [x]_{-1}^{1} = 4\pi.$

Example



$y=\frac{1}{x}$ between 1 and ∞.

1 and t, then let $t\to\infty$.

volume $V=\int_1^t \frac{\pi}{x^2}\,dx = \pi\left[-\frac{1}{x}\right]_1^t = \pi\left(1-\frac{1}{t}\right)$

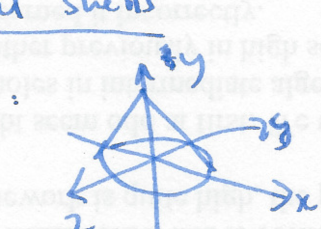
$\to \pi$ as $t\to\infty$.

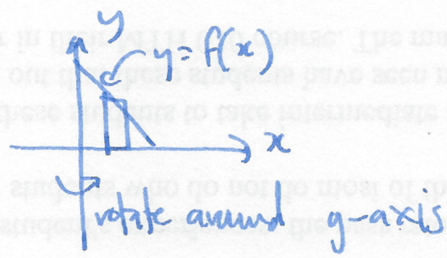
surface area $A = 2\pi\int_1^t \frac{1}{x}\sqrt{1+\frac{1}{x^4}}\,dx > 2\pi\int_1^t \frac{1}{x}\,dx = 2\pi\left[\ln|x|\right]_1^t$

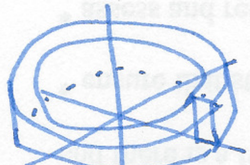
$= 2\pi\ln(t)\to\infty$ as $t\to\infty$

§6.4 Cylindrical shells

Volume of cone:





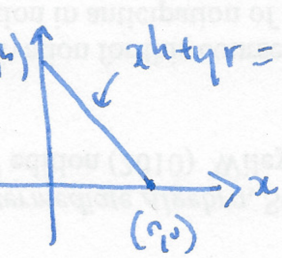


volume of shell $V_i \approx 2\pi x y \Delta x$

total volume $V \approx \sum V_i = \sum 2\pi x_i y(x_i)\Delta x_i$

$$V = 2\pi\int_a^b x f(x)\,dx$$

Example



$hx+yr=hr$

$$V=\int_0^r 2\pi x\left(\frac{hr-xh}{r}\right)dx$$

$$V = \frac{2\pi h}{r}\int_0^r x(r-x)\,dx = 2\pi\frac{h}{r}\int_0^r xr-x^2\,dx = 2\pi\frac{h}{r}\left[\frac{1}{2}x^2r-\frac{1}{3}x^3\right]_0^r$$

$$= 2\pi\frac{h}{r}\left(\frac{1}{2}r^3-\frac{1}{3}r^3\right) = \frac{1}{3}\pi h r^2$$