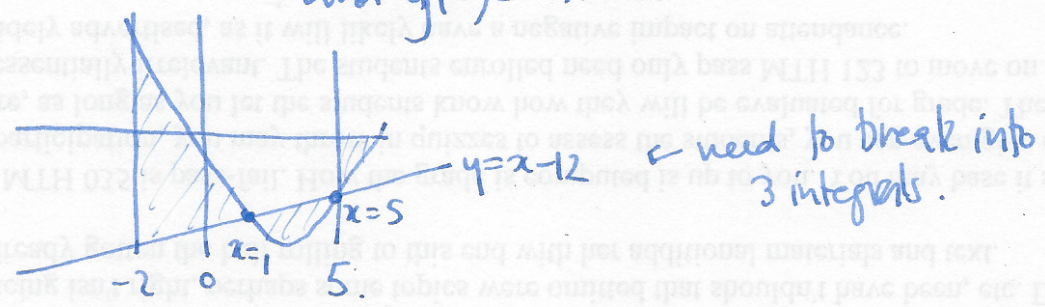


Examples

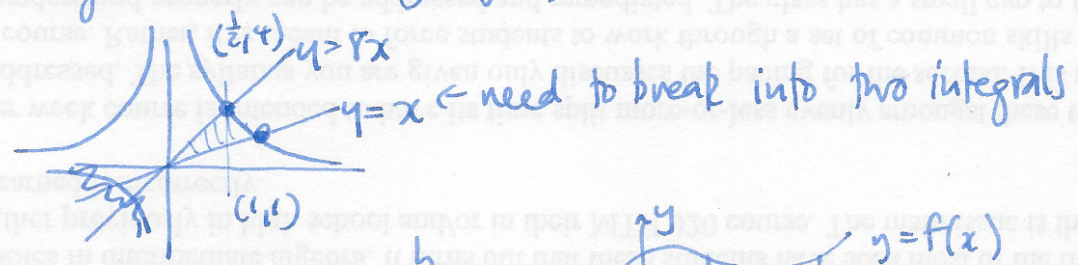
- find area between the graphs $f(x) = x^2 - 5x - 7$ on $[-2, 5]$ and $g(x) = x - 12$

draw picture:



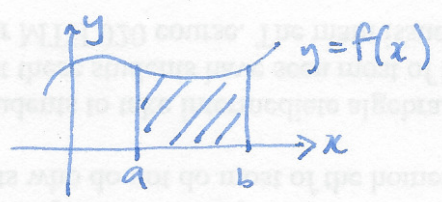
- find area of region bounded by $y = \frac{1}{x^2}$, $y = x$, $y = 8x$

draw picture:

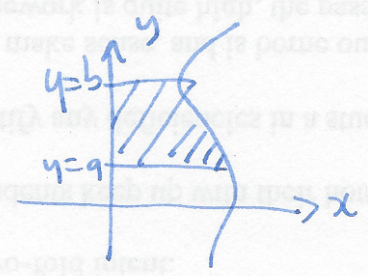


Integration along y-axis

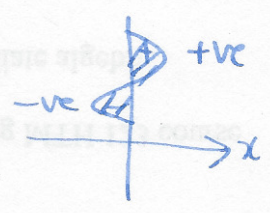
recall: $\int_a^b f(x) dx$



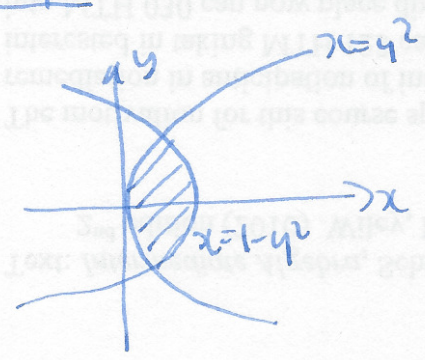
$$\int_a^b f(y) dy$$



signed area:

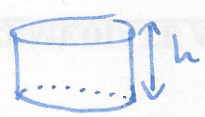


Example find area between the curves $x = y^2$ and $x = 1 - y^2$



§6.2 Volume, average value

recall: volume of cylinder is Ah



A = area of base

fact: this works for cylinders of any base shape:



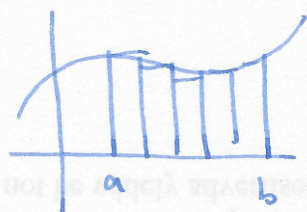
$$V = Ah$$

suppose the shape is not a cylinder:



can approximate by horizontal slices

Recall:



← can approximate area under curve by rectangles of width Δx .



← can approximate volume of object by cylinders of width Δy say

$$\text{Volume } V = \sum V_i = \sum A(y_i) \Delta y$$

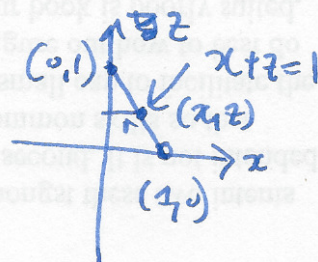
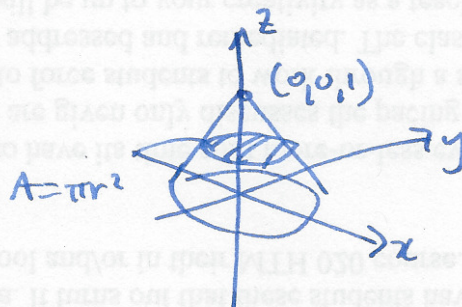
volume

$$V = \int_a^b A(y) dy$$

Example Find volume of cone:

$$V = \int_0^1 A(z) dz$$

$$= \int_0^1 \pi (1-z)^2 dz = \pi \int_0^1 1 - 2z + z^2 dz = \pi \left[z - z^2 + \frac{1}{3} z^3 \right]_0^1 = \pi \left(1 - 1 + \frac{1}{3} \right) = \frac{\pi}{3}$$

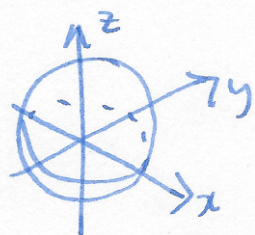


in general: $\frac{x}{r} + \frac{z}{h} = 1 \quad x = \left(1 - \frac{z}{h}\right)r$

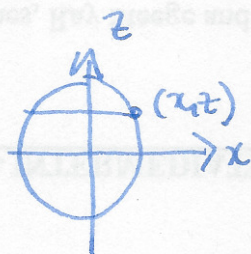
$$V = \int_0^h A(z) dz = \int_0^1 \pi r^2 \left(1 - \frac{z}{h}\right)^2 dz = \pi r^2 \int_0^1 1 - \frac{2z}{h} + \frac{z^2}{h^2} dz$$

$$= \pi r^2 \left[z - \frac{z^2}{h} + \frac{z^3}{3h^2} \right]_0^h = \pi r^2 \left(h - \frac{h^2}{h} + \frac{h^3}{3h^2} \right) = \frac{1}{3} \pi r^2 h$$

Example Volume of sphere



$$\int_{-r}^r A(z) dz$$



$$x^2 + z^2 = r^2$$

$$x = \sqrt{r^2 - z^2}$$

$$= 2 \int_0^r 2\pi (r^2 - z^2) dz = 2\pi \left[r^2 z - \frac{1}{3} z^3 \right]_0^r = 2\pi \left(r^3 - \frac{1}{3} r^3 \right) = \frac{4}{3} \pi r^3$$