

useful rules

$f(x)$	$f'(x)$	$f(x)$	$\int f(x) dx$
x^n	nx^{n-1}	x^n	$\frac{x^{n+1}}{n+1} \quad (n \neq -1)$
$\sin(x)$	$\cos(x)$	$\sin(x)$	$-\cos(x)$
$\cos(x)$	$-\sin(x)$	$\cos(x)$	$\sin(x)$
$\tan(x)$	$\sec^2(x)$	$\frac{1}{x} = x^{-1}$	$\ln x $
$\tan^{-1}(x)$	$\frac{1}{1+x^2}$	e^x	e^x
e^x	e^x		
$\ln(x)$	$1/x$		

general rules

f	f'	$\int f dx$
$kf(x)$	$kf'(x)$	$k \int f(x) dx$
$f(x) \pm g(x)$	$f'(x) \pm g'(x)$	$\int f(x) \pm g(x) dx$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$	$\int f(x)g(x) dx$
$f(g(x))$	$f'(g(x)) \cdot g'(x)$	$\int f(x) dx + \int g(x) dx$
f/g	$\frac{gf' - fg'}{g^2}$	

§5.7 Integration by substitution

$$\int_{x=a}^{x=b} f(x) dx, \quad x(u) \quad \leadsto \quad \int_{u=x^{-1}(a)}^{u=x^{-1}(b)} f(x(u)) \frac{dx}{du} du$$

recall $\frac{dx}{du} = \frac{1}{\frac{du}{dx}}$ integration by substitution is "reverse chain rule".

Examples • $\int x \sin(x^2) dx$ try $x^2 = u$ $\frac{du}{dx} = 2x$

$$\leadsto \int x \sin(x^2) \frac{dx}{du} du = \int x \sin(x^2) \frac{1}{2x} du = \int \sin(u) du$$

$$= -\cos(u) + c = -\cos(x^2) + c \quad \text{check!}$$

• $\int e^{4x} dx$ $u=4x$ $\frac{du}{dx}=4$ $\int e^u \frac{1}{4} du = \frac{1}{4} e^u + C = \frac{1}{4} e^{4x} + C$ check! (5)

• $\int \tan \theta d\theta = \int \frac{\sin \theta}{\cos \theta} d\theta$ [hint: $\frac{d}{dx} (\ln(f(x))) = \frac{1}{f(x)} f'(x)$]

try $u = \cos \theta$ $\frac{du}{d\theta} = -\sin \theta$ $\int \frac{\sin \theta}{u} \cdot \frac{-1}{\sin \theta} d\theta = -\int \frac{1}{u} du = -\ln|u| + C$
 $= -\ln|\cos x| + C$ check!

• $\int x\sqrt{x+1} dx$ try $u=x+1$ $\frac{du}{dx}=1$ $\int x\sqrt{u} du = \int (u-1)\sqrt{u} du$
 $= \int u^{3/2} - u^{1/2} du = \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} + C = \frac{2}{5} (x+1)^{5/2} - \frac{2}{3} (x+1)^{3/2} + C$

Definite integrals

• $\int_0^9 x^2 \sqrt{x^3+1} dx$ $u=x^3+1$ $\frac{du}{dx}=3x^2$ $\int_1^9 x^2 \sqrt{u} \frac{1}{3x^2} du = \int_1^9 \frac{1}{3} u^{1/2} du = \left[\frac{2}{9} u^{3/2} \right]_1^9$

§5.8 More functions

inverse trig functions

$\frac{d}{dx} (\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$

$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x) + C$

$\frac{d}{dx} (\tan^{-1}(x)) = \frac{1}{1+x^2}$

$\int \frac{1}{1+x^2} dx = \tan^{-1}(x) + C$

$\frac{d}{dx} (\sec^{-1}(x)) = \frac{1}{|x|\sqrt{x^2-1}}$

$\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1}(x) + C$

Example $\int_0^1 \frac{1}{4+x^2} dx$ try $u=2x$ $\frac{dx}{du} = \frac{1}{2} \Rightarrow \int_0^{1/2} \frac{1}{4+u^2} 2 du$
 $= \frac{1}{2} \int_0^{1/2} \frac{1}{1+u^2} du = \frac{1}{2} [\tan^{-1}(u)]_0^{1/2} = \frac{1}{2} (\tan^{-1}(1/2) - \tan^{-1}(0)) = \frac{1}{2} \tan^{-1}(1/2)$

exponential functions

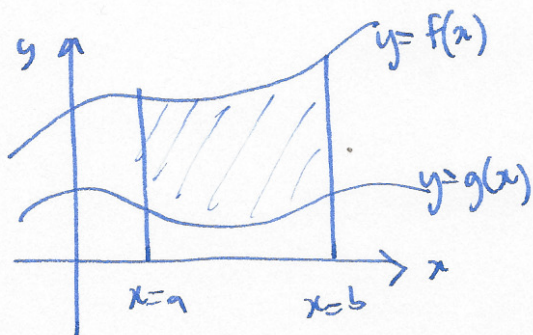
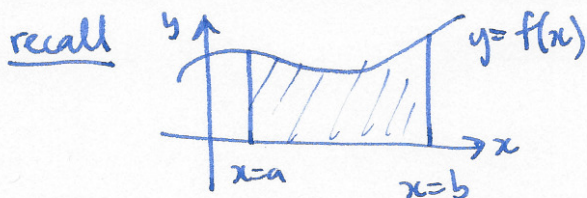
$$\frac{d}{dx}(e^x) = e^x \quad \int e^x dx = e^x + C$$

$$\frac{d}{dx}(b^x) = \frac{d}{dx}(e^{x \ln(b)}) = \ln(b) e^{x \ln(b)} = \ln(b) b^x$$

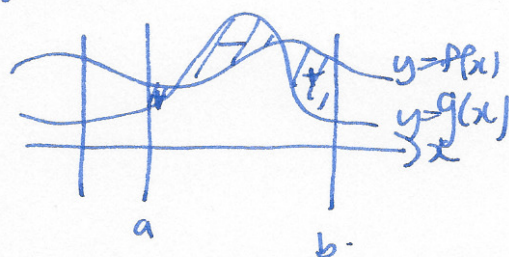
$$\int b^x dx = \frac{1}{\ln(b)} b^x + C$$

Examples $\int_0^1 10^x dx$ $\int_0^{\pi/2} \cos \theta 10^{\sin \theta} d\theta$

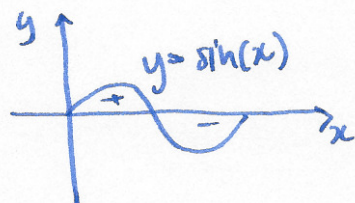
§6.1 Area between two curves



warning: signs!



Q: what do you do if you want "absolute area" (unsigned area)



$$\int_0^{2\pi} \sin(x) dx = 0$$

want: $\int_0^{2\pi} |\sin(x)| dx$ ← need to know when $\sin(x)$ is +ve or -ve.

i.e. need to find zeros: $\sin(x) = 0$

$$\int_0^{\pi} \sin(x) dx + \int_{\pi}^{2\pi} -\sin(x) dx$$

$$= [-\cos(x)]_0^{\pi} - [\cos(x)]_{\pi}^{2\pi}$$

$$= 2 - (-2) = 4$$