MTH 232 Calculus 2

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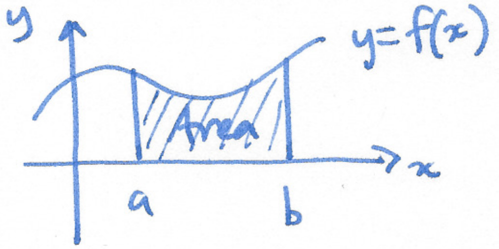
- math tutoring: 1S-214
- students w/ disabilities

Text: Calculus, early transcendentals, Rogawski + Adams

HW: webwork / Matlab projects / quizes

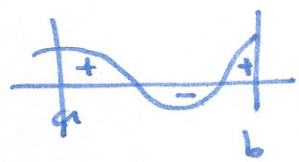
§5.2 Definite Integral



intuition:

$$\int_a^b f(x)\,dx = \text{area under the curve } y=f(x) \text{ between } x=a \text{ and } x=b$$

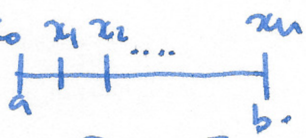
note: signed area



formal defn: Riemann sum $R(f,P,c)$ $\Vert$ $\sum f(c_i)\Delta x_i$ $\quad \Delta x_i = |x_i - x_{i-1}|$

f function

P partition of $[a,b]$

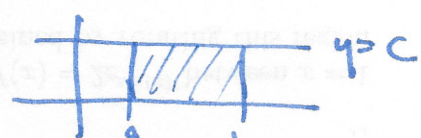


c a choice of point $c_i \in [x_{i-1}, x_i]$

$$\int_a^b f(x)\,dx = \lim_{\Vert P\Vert \to 0} R(f,P,c), \qquad \Vert P\Vert = \max \Delta x_i$$

useful properties

$$\int_a^b c\,dx = c(b-a)$$



$$\int_a^b f(x)+g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$$

reversing limits : $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

0-length interval : $\int_a^a f(x)\,dx = 0$

adjacent intervals : $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

§5.3 Indefinite integrals

Def^{n} A function $F(x)$ is an anti-derivative for $f(x)$ if $F'(x) = f(x)$.

General antiderivative : if $F(x)$ is an anti-derivative for $f(x)$, then any other anti-derivative is of the form $F(x)+c$ for some constant c.

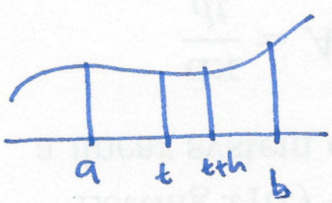
[Proof : suppose f has anti-derivatives F, H. then $(F-H)' = F' - H' = f - f = 0$
$\Rightarrow$ $F - H =$ const function].

notation $\int f(x)\,dx = F(x) + c$ means $F(x)+c$ is the general anti-derivative for $f(x)$.

§5.4 Fundamental theorem of calculus I

Thm (FTC ①) suppose $f(x)$ is cts on $[a,b]$ and $F(x)$ is an anti-derivative for $f(x)$, i.e. $F'(x) = f(x)$. Then $\int_a^b f(x)\,dx = F(b) - F(a)$

intuition :



Consider $\int_a^t f(x)\,dx$ ← function of t!

Q: what is the rate of change wrt to t?

recall $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, so $\frac{d}{dt}\left(\int_a^t f(x)\,dx\right) =$

$$\lim_{h\to 0} \frac{\int_a^{t+h} f(x)\,dx - \int_a^t f(x)\,dx}{h} = \lim_{h\to 0} \frac{\int_t^{t+h} f(x)\,dx}{h} \approx \frac{\overset{\text{area of rectangle}}{f(t)\times h}}{h}$$

$= f(t)$

i.e. $\int_a^t f(x)\,dx$ is an anti-derivative for $f(x)$, so $\int_a^t f(x)\,dx = F(t) + c$

Q: what is the constant? $t=a$: $\int_a^a f(x)\,dx = 0 = F(a) + c \Rightarrow c = -F(a)$

so $\int_a^t f(x)\,dx = F(t) - F(a)$ $\square$.

<u>Example</u> $\int_2^3 \sqrt{x} + \frac{1}{x} + \sin(x)\,dx = \left[\frac{3x^{3/2}}{2} + \ln|x| - \cos(x)\right]_2^3$

$= \frac{3(3)^{3/2}}{2} + \ln(3) - \cos(3) - \left(\frac{3(2)^{3/2}}{2} + \ln(2) - \cos(2)\right)$.

§5.5 Fundamental theorem of calculus II

<u>Thm</u> (FTC 2) let $f(x)$ be a cts function on $[a,b]$, then $A(x) = \int_a^x f(t)\,dt$ is an anti-derivative for $f(x)$, i.e. $A'(x) = f(x) = \frac{dA}{dx} \Leftrightarrow \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.

Furthermore: $A(a) = 0$

<u>Example</u> $\int_0^x e^{-t^2}\,dt$ $\leftarrow$ a function with derivative e^{-x^2}.

<u>Example</u> what about $\int_0^{x^2} \sin(t)\,dt$ $\leftarrow$ function of a function.

to find $\frac{d}{dx}\int_0^{x^2} \sin(t)\,dt$ set $A(x) = \int_0^x \sin(t)\,dt$, then $A'(x) = \sin(x)$

so $\frac{d}{dx}\int_0^{x^2} \sin(t)\,dt = \frac{d}{dx}\left(A(x^2)\right) \underset{\text{chain rule}}{=} A'(x^2)\,(x^2)' = A(x^2).\,2x$
$= \sin(x^2).\,2x$.

[<u>aside</u>: when people say "not every formula can be integrated" what do they mean? If $f(x)$ cts then $A(x) = \int_0^x f(t)\,dt$ is an integral for $f(x)$, but might not be able to write it as a formula including basic functions].

<u>Analogy</u>
- $\sqrt{2} \approx 1.414\ldots$ is a number but not a fraction
- $x^5 - x - 1$ has 1 real root which cannot be written as an expression involving rational numbers and fractional powers. [Galois theory]
- e^{-x^2} has an integral that can't be written as a combination of elementary functions [differential Galois theory].