Math 232 Calculus 2 Fall 17 Midterm 1b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Find $\displaystyle\int \frac{e^{-3x}}{e^{-3x}+1}\,dx$.

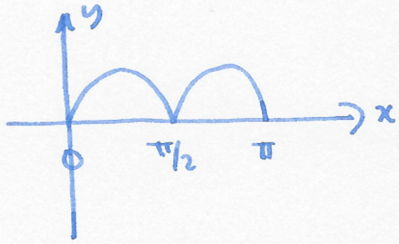
$u = e^{-3x}$

$\frac{du}{dx} = -3e^{-3x}$

$$\int \frac{e^{-3x}}{u+1} \cdot \frac{dx}{du}\,du = \int \frac{e^{-3x}}{u+1} \cdot \frac{1}{-3e^{-3x}}\,du$$

$$= -\frac{1}{3}\int \frac{1}{u+1}\,du = -\frac{1}{3}\ln|u+1| + C = -\frac{1}{3}\ln\left|e^{-3x}+1\right| + C$$

(2) (10 points) Find $\int_0^{\pi} |\sin(2x)|\ dx$. Draw a picture of the region.

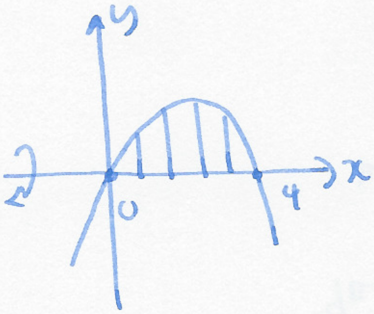


$$\int_0^{\pi/2} \sin 2x\, dx - \int_{\pi/2}^{\pi} \sin 2x\, dx$$

$$= \left[-\tfrac{1}{2}\cos 2x\right]_0^{\pi/2} - \left[-\tfrac{1}{2}\cos 2x\right]_{\pi/2}^{\pi}$$

$$= -\tfrac{1}{2}(-1) + \tfrac{1}{2}(1) \quad + \tfrac{1}{2}(1) - \tfrac{1}{2}(-1) \quad = 2$$

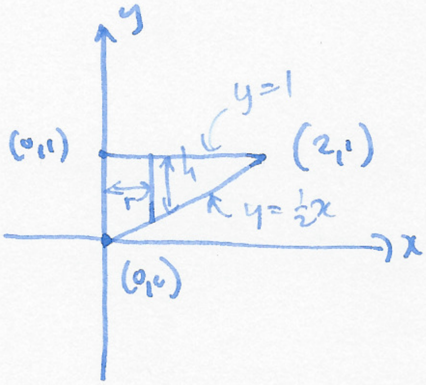
(3) (10 points) Draw a picture of the region bounded by the curve $y = 4x - x^2$, for $y \geq 0$. Write down an integral to give you the volume of revolution of this region about the x-axis. DO NOT EVALUATE THIS INTEGRAL.



discs : $V = \int_0^4 \pi r^2 \, dx$

$= \int_0^4 \pi (4x - x^2) \, dx$

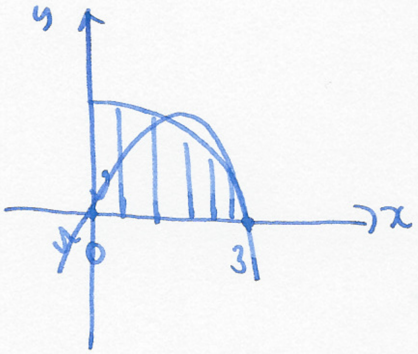
(4) (10 points) Use shells to write down an integral for the volume of the cone formed by rotating the triangle with vertices $(0,0)$, $(0,1)$ and $(2,1)$ about the y-axis. DO NOT EVALUATE THIS INTEGRAL.



shells: $V = \int_0^2 2\pi r h \, dx$

$= \int_0^2 2\pi x \left(1 - \frac{1}{2}x\right) dx$

(5) (10 points) Consider the subset of the plane bounded by $y = 9 - x^2$ in the first quadrant (i.e. $x \geq 0$ and $y \geq 0$). Find the volume of revolution of the 3-dimensional shape formed by rotating this region around the x-axis.



$$V = \int_0^3 \pi r^2 \, dx$$

$$= \int_0^3 \pi (9-x^2)^2 \, dx$$

$$= \pi \int_0^3 81 - 18x^2 + x^4 \, dx = \pi \left[81x - 6x^3 + \frac{1}{5}x^5 \right]_0^3$$

$$= \pi \left(243 - 6 \times 27 + \frac{1}{5} 3^5 \right) = \text{[illegible]}$$

$$\frac{648}{5}\pi \simeq 407.15$$

$$\int u v' \, dx = uv - \int u' v \, dx$$

(6) (10 points) Find $\int \underbrace{\sqrt[3]{x}}_{v'}\, \underbrace{\ln(2x)}_{u} \, dx$.

$u = \ln(2x)$ $\quad u' = \frac{1}{2x} \cdot 2 = \frac{1}{x}$

$v' = x^{1/3}$ $\quad V = \frac{3}{4} x^{4/3}$

$$= \frac{3}{4} x^{4/3} \ln(2x) - \int \frac{3}{4} x^{4/3} \cdot \frac{1}{x} \, dx$$

$$= \frac{3}{4} x^{4/3} \ln(2x) - \int \frac{3}{4} x^{1/3} \, dx$$

$$= \frac{3}{4} x^{4/3} \ln(2x) - \frac{9}{16} x^{4/3} + C$$

$$\int u'v\,dx = uv - \int uv'\,dx$$

(7) (10 points) Find $\int \underbrace{e^{-2x}}_{v} \underbrace{\sin(x)}_{u'}\, dx$.

$v = e^{-2x}$ $\quad v' = -2e^{-2x}$

$u' = \sin x$ $\quad u = -\cos x$

$$\int e^{-2x} \sin x\, dx = -e^{-2x}\cos x - \int \underbrace{2e^{-2x}}_{v} \underbrace{\cos x}_{u'}\, dx$$

$v = 2e^{-2x}$ $\quad v' = -4e^{-2x}$

$u' = \cos x$ $\quad u = \sin x$

$$\int e^{-2x} \sin x\, dx = -e^{-2x}\cos x - 2e^{-2x}\sin x + \int -4e^{-2x}\sin x$$

$$\int e^{-2x} \sin x\, dx = \frac{1}{5}\left(-e^{-2x}\cos x - 2e^{-2x}\sin x\right) + c$$

(8) Find $\int_0^{\pi/2} \cos^3 x \, dx$. $= \int_0^{\pi/2} \cos x \left(1 - \sin^2 x\right) dx$ $\quad u = \sin x$

$\frac{du}{dx} = \cos x$

$= \int_0^{\cancel{\pi/2}\,1} \cos x \left(1-u^2\right) \frac{dx}{du} \, du = \int_0^{\cancel{\pi/2}\,1} \cos x \left(1-u^2\right) \frac{1}{\cos x} \, du = \int_0^1 1 - u^2 \, du$

$= \left[u - \frac{1}{3}u^3\right]_0^1 = 1 - \frac{1}{3} = \frac{2}{3}$.

(9) Find $\int \cos(4x)\cos(3x)\, dx$.

$$\left.\begin{aligned} \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ \cos(A-B) &= \cos A \cos B + \sin A \sin B \end{aligned}\right\} \quad \cos A \cos B = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$$

$$= \int \frac{1}{2}\cos(\overset{7x}{\cancel{A+B}}) + \frac{1}{2}\cos(x)\, dx = \frac{1}{14}\sin(7x) + \frac{1}{2}\sin(x) + C$$

(10) Find $\int \frac{1}{4+x^2}\,dx.$ $= \frac{1}{4}\int \frac{1}{1+x^2/4}\,dx$

$u = x/2$

$\frac{du}{dx} = 1/2.$

$= \frac{1}{4}\int \frac{1}{1+u^2}\cdot 2\,du = \frac{1}{2}\int \frac{1}{1+u^2}\,du = \frac{1}{2}\tan^{-1}(u)+c = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+c$