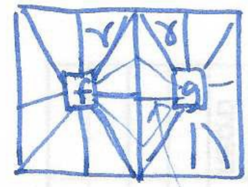
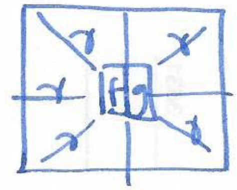
$[\gamma f] + [\gamma g] = [\gamma(f+g)]$





this path is $\bar\gamma\gamma$ homotopic to x_0.

inverse ~~isomorphism~~ homo: $\gamma^{-1}: \pi_n(X, x_0) \to \pi_n(X, x_1)$

with $\gamma\gamma^{-1} = id$ $\quad \bar\gamma\gamma f = id \Rightarrow \gamma, \gamma^{-1}$ isos. $\square$.

π_n is a functor: $\quad \varphi: (X, x_0) \to (Y, y_0)$

induces $\quad \varphi_*: \pi_n(X, x_0) \to \pi_n(Y, y_0)$.

$$f: (S^n, s_0) \to (X, x_0) \longmapsto \varphi \cdot f: (S^n, s_0) \to (X, x_0) \to (Y, y_0)$$

$$\varphi_*(f) = \varphi \cdot f$$

$Top_0 = \{$topological spaces w/ basepoints (X, x_0) and basepoint preserving maps$\}$.

$\pi_n: Top_0 \to$ Abelian groups.

<u>Prop^n</u> If $\varphi: (X, x_0) \to (Y, y_0)$ is a homotopy equivalence, then φ_* is an isomorphism on π_n for all n. $\square$.

<u>Corollary</u> If X is contractible then $\pi_n X = 0$ for all n.

<u>Fact</u> If X is a finite CW-complex/simplicial complex, then all $\pi_n X$ known iff $\tilde X$ is contractible.

<u>Prop^n</u> A covering map $p: (\tilde X, \tilde x_0) \to (X, x_0)$ induces an isomorphism on π_n for all $n \geq 2$.

<u>Corollary</u> $\pi_n(T^k) = 0$ for all $n \geq 2$.

<u>Proof</u> consider $p_*: \pi_n(\tilde X, \tilde x_0) \to \pi_n(X, x_0)$

$$\begin{tikzcd} & (\tilde X, x_0) \arrow[d, "p"] \\ (S^n, s_0) \arrow[ur, "\tilde f"] \arrow[r, "f"'] & (X, x_0) \end{tikzcd}$$

<u>onto</u>: $\pi_1 S^n = \{1\}$, so every map lifts, $p_*([\tilde f]) = [f]$

injective: homotopy lifting property $\square$.

Thm [Whitehead] If a map $f: X \to Y$ between connected CW-complexes induces isomorphisms $f_*: \pi_n X \to \pi_n Y$ for all n, then f is a homotopy equivalence. If f is an inclusion of subcomplexes, then X is a deformation retract of Y.

Thm [Freudenthal suspension theorem] The suspension map $\pi_i(S^n) \to \pi_{i+1}(S^{n+1})$ is an iso for $i < 2n-1$ and a surjection for $i = 2n-1$. (holds for X CW-complex which is $(n-1)$-connected).

Corollary $\pi_n(S^n) \cong \mathbb{Z}$ in particular, the degree map $\pi_n(S^n) \to \mathbb{Z}$ is an iso.

$$\pi_1(S^1) \xrightarrow{\text{surjective}} \pi_2(S^2) \xrightarrow{\text{iso}} \pi_3(S^3) \xrightarrow{\text{iso}} \cdots$$

just have to show $\pi_2(S^2)$ is infinite, degree does this. $\square$.

Thm [Hurewicz] If X is $(n-1)$-connected, $n \geq 2$, then $\tilde{H}_i(X) = 0$ for $i < n$ and $\pi_n X \cong H_n(X)$.

Thm [Homology Whitehead] If $f: X \to Y$ is a map between CW-complexes induces $f_*: H_n X \to H_n(Y)$ isos for all n, then f is a homotopy equivalence.