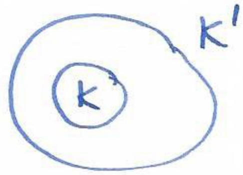
X top space

$I = \{\text{compact subsets of } X\}$, $\leq$ = containment of sets.

$G_K = H^n(X|K)$



$$\begin{matrix}(X, X\setminus K') \\ \downarrow \\ (X, X\setminus K)\end{matrix}$$

if $K \subseteq K'$ then $\underbrace{H^n(X|K) \xrightarrow{i^*} H^n(X|K')}_{}$

typically not surjective or injective

<u>Prop^n:</u> $H^n_c(X) = \varinjlim_{K \text{ compact}} H^n(X|K)$

<u>Proof</u> we have maps, for each K, $H^n(X|K) \longrightarrow H^n_c(X)$

as $\phi \in C^n(X|K)$ vanishes on σ in $X \setminus K$.

this is compatible with inclusions, so get $\varinjlim_{K \text{ compact}} H^n(X|K) \longrightarrow H^n_c(X)$

<u>surjective:</u> $[\phi] \in H^n_c(X)$ lies in the image of $H^n(X|K_\phi)$

<u>injective:</u> if $[\phi] \in H^n(X|K)$ is 0 in $H^n_c(X)$, then $\phi = \delta\psi$ where

ψ is supported on $K_\psi \supseteq K_\phi \cup K$, so $[\phi] = 0$ in $H^n(X|K_\psi \cup K_\phi)$ $\square$.

<u>Remark</u> : don't need to use all compact sets,
just enough s.t. every compact set is contained in some K_α.

<u>Example</u> $X = \mathbb{R}^n$ $\quad B_k = B_k^{\text{open}}(0)$ $\quad H^*_c(\mathbb{R}^n) = \varinjlim \underbrace{H^*(\mathbb{R}^n|B_k)}_{H^*(\mathbb{R}^n/\mathbb{R}^n \setminus B_k)} = \tilde{H}^*(S^n)$

<u>Remark</u> H^*_c <u>not</u> a homotopy invariant!

$\mathbb{R}^n \simeq \mathbb{R}^m$ but $H^i_c(\mathbb{R}^m) = \begin{cases} \mathbb{Z} & i = m \\ 0 & \text{else} \end{cases}$