

Proof $\chi(M) = \sum (-1)^i \text{rank}(H_i(M; \mathbb{Z}))$

$= \sum (-1)^i \dim(H_i(M; \mathbb{Z}_2))$

n odd, then i and $n-i$ have opposite parity, so cancel in pairs.

⊛: If M is a finite cell complex, then both are $\sum (-1)^i (\# \text{ of } i \text{ cells})$

if M has finitely generated $H_i(M; \mathbb{Z})$ then use UCT:

$$\dim H_i(M; \mathbb{Z}_2) = \text{rank } H_i(M; \mathbb{Z}) + \text{rank 2-part of } H_i(M; \mathbb{Z}) + \text{rank 2-part of } H_{i-1}(M; \mathbb{Z})$$

Fact: any closed n -manifold is homotopy equivalent to a CW-complex (in fact if $n \neq 4$ then M^n has a CW-complex structure). (Hatcher A.12)

Dual triangulations

Let M be a manifold with a triangulation T , then there is a dual cell decomposition D

	k -simplex in T	$n-k$ cell in D
--	---------------------	-------------------

$\sigma \longleftrightarrow \hat{\sigma}$

reverses inclusion relations: $\sigma_0 < \sigma_1 \Rightarrow D(\sigma_0) > D(\sigma_1)$

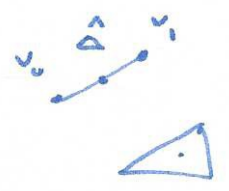
↑
subsimplex/cell

aim: get an isomorphism of chain complexes:

cohomology complex of T
 $C^* \longrightarrow D_*$
homology complex of D .

this proves Poincaré duality for triangulated manifolds.

simplex $[v_0, \dots, v_n] \subset \mathbb{R}^m$. barycenter is $\hat{\Delta} = \frac{1}{n+1} \sum v_i$



recall: barycentric subdivision

$$s(\Delta) = \bigcup \{ [\hat{\Delta}, w_0, \dots, w_{n-1}] \mid [w_0, \dots, w_{n-1}] \text{ simplex in } s(\partial\Delta) \}$$