(i.e
$$\begin{array}{ccccc} C_{n+1} & \xrightarrow{\partial} & C_n & \xrightarrow{\partial} & C_{n-1} \\ & \delta\phi \searrow & \downarrow \phi & & \\ & & G & & \end{array}$$

check: chain complex $\quad \delta_n \delta_{n-1}(\phi) = \delta_n(\phi \circ \partial_n) = \phi \circ \underbrace{\partial_n \circ \partial_{n-1}}_{0} = 0.$

<u>Defn</u> $H^n(X;G) = \ker(\delta_n) / \operatorname{im} \delta_{n-1}$ (homology groups of the cochain complex)

<u>Note</u>: $X \xrightarrow{f} Y$

$C_n(X) \xrightarrow{f_*} C_n(Y)$

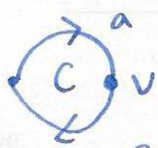
$C^n(X) \xleftarrow{f^*} C_n(Y) \qquad f^*(\phi : C_n(Y) \to G) = \phi \circ f_*$

gives map on H^n as f^* is a chain map.

<u>Example</u> S^1

$$\left.\begin{array}{ll} C_*(S^1;\mathbb{Z}) & 0 \to \mathbb{Z} \xrightarrow{0} \mathbb{Z} \to 0 \\ C^*(S^1;\mathbb{Z}) & 0 \leftarrow \mathbb{Z} \xleftarrow{0} \mathbb{Z} \leftarrow 0 \end{array}\right\} \quad H^k(S^1;\mathbb{Z}) = \begin{cases} \mathbb{Z} & k = 0,1 \\ 0 & \text{else.} \end{cases}$$

<u>Example</u> $\mathbb{RP}^2$



$$\begin{array}{lcccccccc} C_* : & 0 \to & \overset{2}{\mathbb{Z}} & \xrightarrow[\times 2]{} & \overset{1}{\mathbb{Z}} & \xrightarrow[0]{} & \overset{0}{\mathbb{Z}} & \to 0 \\ H_* : & & 0 & & \mathbb{Z}/2 & & \mathbb{Z} & \end{array}$$

$$C^* : \quad 0 \leftarrow \mathbb{Z} \xleftarrow{\times 2} \mathbb{Z} \xleftarrow{0} \mathbb{Z} \leftarrow 0$$

$\langle \alpha \rangle \quad \langle \phi \rangle$

$\alpha(c) = 1 \quad \phi(a) = 1$

$H^* : \quad \mathbb{Z}/2 \quad 0 \quad \mathbb{Z}$

$\delta_1(\phi)(c)$

$= \phi(\partial_2 c)$

$= \phi(2a) = 2$

$\Rightarrow \delta_1(\phi) = 2\alpha.$