(1) Find a vector field on S^2 with exactly one zero.

(2) Describe explicit cell structures on the following spaces, and then use cellular homology to compute their homology groups.

   (a) The Möbius band.

   (b) The solid torus $S^1 \times D^2$.

   (c) The space formed by gluing the boundary of a disc to a figure eight curve in the interior of another disc; more precisely choose an embedding of $S^1 \vee S^1$ in the interior of the unit disc, and let a and b be the standard generators for $\pi_1(S^1 \vee S^1)$. Then glue in another disc along its boundary using the gluing map ab.

(3) Use the Mayer-Vietoris theorem to compute the homology of the following spaces.

   (a) The space formed by gluing the boundary of a Möbius band by a homeomorphism to an essential curve on a torus $S^1 \times S^1$.

   (b) Let A be the unit disc with the interior of two disjoint open discs removed, and let $h\colon A \to A$ be an orientation preserving homeomorphism which swaps the two holes. Let X be $A \times I/\sim$ where $(x, 1) \sim (h(x), 0)$; equivalently, take a solid torus and remove a small neighbourhood of an unknotted curve which loops around the core curve twice. Compute the homology of X by applying the Mayer-Vietoris theorem to the solid torus as the union of X and the removed solid torus. You should get the same asnwer as in (a), why is this?

   (c) The space Y formed by taking two copies of X and gluing their boundaries together by the identity homeomorphism.

Also: Hatcher §2.2 Q29