Math 229 Calculus Computer Lab Spring 16 Midterm 2b

Name: _____

- I will count your best 8 of the following 10 questions.
- You may only use `julia` during this exam. No calculators or cell phones or notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(1) Convert the following `julia` expressions to standard mathematical expressions.

(a) `1/4*x+1/x/2`

$$\frac{x}{4} + \frac{1}{2x}$$

(b) `1/3x-e^x`

$$\frac{1}{3x} - e^x$$

(2) Convert the following mathematical expressions to `julia` expressions.

(a) $\cos^3(\sqrt[3]{(x)})/2$

```
cos( x^(1/3))^3 /2
```

(b) $\sin^{-1}(\frac{1}{3}x)$

```
asin(x/3)
```

(3) Find all solutions (to 3 decimal places) to the equation $10\sin(3x) = -6x+300$. Write down the `julia` command you use.

$f(x) = 10\sin(3x) + 6x - 300$

```
plot(f, -100, 100)
plot(f, 40, 60)
can see 3 roots
fzeros(f, 45, 55)
```

48.5308
49.005
50.2211

(4) Consider the equation $e^{x/3} = 6/x$.

(a) Show there is a solution by plotting the graphs of these functions. List the commands you use.

(b) Write `julia` commands to find a numerical approximation to the solution, and find the solution.

a) $f(x) = \exp(x/3) - 6/x$

```
plot(f,-10,10)
plot(f,1,10)
```

solution approx 3.

b) `fzeros(f,2,4)`

2.55782

(5) Find the location of the local maxima of $f(x) = -e^{x/70} - e^{-x/7}$ to two decimal places. Write out the `julia` commands you use.

```
f(x) = -exp(x/70)-exp(-x/7)
plot(f,-50,500)
0,200
ctz
```

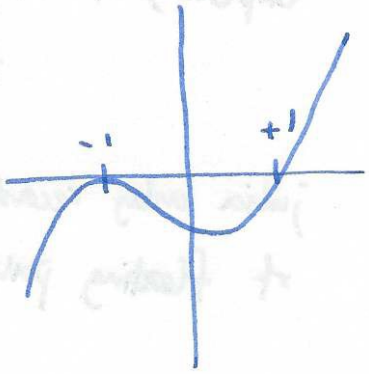
max at $x = 14.65$

(6) Use the bisection method `fzero` to solve the previous question, i.e. find the zeros of $f(x) = x^3 + x^2 - x - 1$. Write down the `julia` commands you use. How many roots do you find? Explain.

$f(x) = x^3 + x^2 - x - 1$

plot(f, -2, 2)

roots(f) 1
-1
-1



fzero(f, -2, 2) only finds one root at $x = +1$
as bisection method can't find double roots
as needs an interval with endpoints taking f values
of opposite sign.

(7) Use `julia` to evaluate `exp(100)+1-exp(100)`. Explain `julia`'s answer.

exp(100) + 1 - exp(100) = 0

floating point rounding error —

julia only records first ≈ 16 decimal places of floating point numbers

(8) You wish to estimate

$$\lim_{x \to 0} \frac{e^{2x} - 1}{\sin(3x)}.$$

Write `julia` commands to generate a list of numbers $\{10^{-1}, 10^{-2}, \ldots, 10^{-10}\}$. Evaluate the function when x takes these values, and write down your results. What do you think the limit is? Explain `julia`'s output.

```
xs = map(x -> 10.0^-x, collect(1:10))
f(x) = (exp(2x)-1)/sin(3x)
map
map(f, xs)
```

```
0.749197
0.673479
0.667335
0.666733
0.666673
0.666667
  ⋮
0.666667
```

limit is $\frac{2}{3}$.

(9) Use `julia` to estimate $\lim_{x \to 0} \frac{\tan(3x)}{2xe^{2x}}$ by any method. Write down the `julia` commands you use.

limit is $\frac{3}{2}$, any method

(10) Use `julia` to estimate $\lim_{x \to 0} \dfrac{\tan(3x) - 3x}{\sin^3(x/3)}$ by any method. Write down the `julia` commands you use.

limit is 243, any method