Name: ______________________

- I will count your best 8 of the following 10 questions.
- You may only use Julia during this exam. No calculators or cell phones or notes.

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| 80 |
(1) Convert the following Julia expressions to standard mathematical expressions.

(a) \( \frac{x}{4} + \frac{1}{2x} \)

(b) \( \frac{1}{3x} - e^{-x} \)
(2) Convert the following mathematical expressions to julia expressions.

(a) \( \cos^3(\sqrt[3]{x})/2 \)

\[ \cos \left( x^{\left(\frac{1}{3}\right)}\right)^3 \cdot \frac{1}{2} \]

(b) \( \sin^{-1}(\frac{1}{3}x) \)

\[ \arcsin \left( \frac{x}{3} \right) \]
(3) Find all solutions (to 3 decimal places) to the equation $10 \sin(3x) = -6x + 300$. Write down the Julia command you use.

\[
f(x) = 10 \sin(3x) + 6x - 300
\]

\[
\text{plot}(f, -100, 100)
\]

\[
\text{plot}(f, 40, 60)
\]

Can see 3 roots

\[
\text{roots}(f, 45, 55)
\]

49.5308
49.005
50.2211
(4) Consider the equation $e^{x/3} = 6/x$.

(a) Show there is a solution by plotting the graphs of these functions. List the commands you use.

(b) Write Julia commands to find a numerical approximation to the solution, and find the solution.

\[ f(x) = \exp(x/3) - 6/x \]

\[ \text{plot } (f, -10, 10) \]

\[ \text{plot } (f, 1, 10) \]

\[ \text{solve for } x \approx 3 \]

\[ f(2.5) = 0 \approx 2.5578 \]
(5) Find the location of the local maxima of \( f(x) = -e^{x/70} - e^{-x/70} \) to two decimal places. Write out the Julia commands you use.

\[
f(x) = -\exp(x/70) - \exp(-x/70)
\]

```
plot(f, -50:50)
  0, 200
0.25
```

max at \( x = 14.65 \)

```
(f(x) = exp(x/20) - exp(-x/20) , (0, 0, 0), t = 1, 1, 1, 1, 1)
```

```
sumdiff approx?
```
(6) Use the bisection method \texttt{fzero} to solve the previous question, i.e. find the zeros of \( f(x) = x^3 + x^2 - x - 1 \). Write down the \texttt{Julia} commands you use. How many roots do you find? Explain.

\[
f(x) = x^3 + x^2 - x - 1
\]

\[
\text{plot}(f, -2, 2)
\]

\[
\text{roots}(f)
\]

\[
\text{fzero}: (f, -2, 2) \text{ only finds one root at } x = -1
\]

\[
\text{as bisection method can't find double roots } \checkmark
\]

\[
\text{as needs an interval with endpoints taking } f \text{ values of opposite sign}
\]
(7) Use julia to evaluate \( \exp(100) + 1 - \exp(100) \). Explain julia's answer.

\[
\exp(100) + 1 - \exp(100) = 0
\]

- floating point rounding error

julia only records first 16 decimal places of floating point numbers
(8) You wish to estimate\[
\lim_{x \to \infty} \frac{e^{2x} - 1}{\sin(3x)}.
\]
Write Julia commands to generate a list of numbers \(\{10^{-1}, 10^{-2}, \ldots, 10^{-10}\}\). Evaluate the function when \(x\) takes these values, and write down your results. What do you think the limit is? Explain Julia's output.

\[
\begin{align*}
x_5 &= \text{map}(x \to 10 \cdot 0.1^x, \text{collect}(1:10)) \\
f(x) &= (\exp(2x) - 1) / \sin(3x) \\
\text{map} \\
\text{map}(f, x_5) \\
&= 0.749197 \\
&= 0.673479 \\
&= 0.667335 \\
&= 0.666733 \\
&= 0.666673 \\
&= 0.666667 \\
&= 0.666667
\end{align*}
\]

\text{limit is } \frac{2}{3}.
(9) Use Julia to estimate \( \lim_{x \to 0} \frac{\tan(3x)}{2x e^{2x}} \) by any method. Write down the Julia commands you use.

\[
\text{\textit{Limit:}} \quad \frac{3}{2}
\]
(10) Use Julia to estimate \( \lim_{x \to 0} \frac{\tan(3x) - 3x}{\sin^3(x/3)} \) by any method. Write down the Julia commands you use.

\[ \text{Limit is 243, any method} \]