Math 229 Calculus Computer Lab Spring 16 Midterm 2a

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may only use `julia` during this exam. No calculators or cell phones or notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(1) Convert the following `julia` expressions to standard mathematical expressions.

(a) `1/x/4+1/2*x`

$$\frac{1}{4x} + \frac{x}{2}$$

(b) `e^x-1/2x`

$$e^x - \frac{1}{2x}$$

(2) Convert the following mathematical expressions to `julia` expressions.

(a) $\sin^3(\sqrt[3]{(x)})/3$

```
sin( x^(1/3) )^3 /3
```

(b) $\cos^{-1}(\frac{1}{2}x)$

```
acos (x/2)
```

(3) Find all solutions (to 3 decimal places) to the equation $10\cos(3x) = -6x + 300$. Write down the `julia` command you use.

f(x) = 10 cos(3x) + 6x - 300

plot(f, -100, 100)

plot(f, 40, 60)

can see 3 roots

fzeros(f, 45, 55)

49.785
51.0048
51.4744

(4) Consider the equation $e^{x/5} = 4/x$.

(a) Show there is a solution by plotting the graphs of these functions. List the commands you use.

(b) Write `julia` commands to find a numerical approximation to the solution, and find the solution.

a)

```
f(x) = exp(x/5) - 4/x
plot(f, -10, 10)
plot(f, 1, 10)
```

solution ≈ 2.5

b)

```
## fzeros(f, 2, 4)
```

2.45034

(5) Find the location of the local maxima of $f(x) = -e^{x/7} - e^{-x/70}$ to two decimal places. Write out the `julia` commands you use.

f(x) = -exp(x/70) - exp(-x/70)

plot(f, -30, 10)

-20, -10

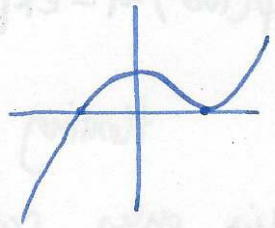
etc.

max at x = -14.65

(6) Use the bisection method `fzero` to ~~solve the previous question, i.e.~~ find the zeros of $f(x) = x^3 - x^2 - x + 1$. Write down the `julia` commands you use. How many roots do you find? Explain.

$f(x) = x^3 - x^2 - x + 1$

plot(f, -2, 2)



roots(f) -1
$+1$
$+1$

fzero(f, -2, 2) only finds 1 root at $x = -1$

as bisection method needs interval with endpoints with values of opposite sign, so can't find double root at $+1$.

(7) Use `julia` to evaluate `exp(100)+1-exp(100)`. Explain `julia`'s answer.

exp(100) + 1 − exp(100) = 0

floating point rounding error—

julia only records first ≃ 16 decimal places of floating point numbers.

(8) You wish to estimate

$$\lim_{x \to 0} \frac{e^{3x} - 1}{\sin(2x)}.$$

Write `julia` commands to generate a list of numbers $\{10^{-1}, 10^{-2}, \ldots, 10^{-10}\}$. Evaluate the function when x takes these values, and write down your results. What do you think the limit is? Explain `julia`'s output.

```
xs = map(x -> 10.0^-x, collect(1:10))

f(x) = (exp(3x) - 1)/sin(2x)

map(f, xs)
```

```
1.76101
1.52283
1.50225
1.50023
1.50002
1.5
 ⋮
1.5
```

limit is $\frac{3}{2}$

(9) Use `julia` to estimate $\lim_{x\to 0} \dfrac{\tan(2x)}{3xe^{2x}}$ by any method. Write down the `julia` commands you use.

limit is $\frac{2}{3}$, any method

(10) Use `julia` to estimate $\lim_{x\to 0} \dfrac{\tan(2x) - 2x}{\sin^3(x/2)}$ by any method. Write down the `julia` commands you use.

limit is $\frac{64}{3} \approx 21.333$, any method