

## Math 229 Calculus Computer Lab Spring 16 Final b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may only use Julia during this exam. No calculators or cell phones or notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Final	
Overall	

- (1) Convert the following julia expressions to standard mathematical expressions. Use parentheses if necessary to clearly indicate the order of operations:

(a)  $x - z / 2x + y$

$$x - \frac{z}{2x} + y$$

(b)  $\cos(2x^2) / x / 2$

$$\frac{\cos(2x^2)}{2x}$$

(c)  $a + c / (b - a)$

$$a + \frac{c}{b - a}$$

	Final
	Overall

(2) Convert the following standard mathematical expressions into julia expressions.

(a)  $\sin^2(\frac{1}{3}x)$

$$\sin(x/3)^2$$

(b)  $e^{-2x^2}$

$$\exp(-2x^2)$$

(c)  $\frac{1}{\sqrt[3]{1+\frac{1}{1-x}}}$

$$1/\text{cbrt}(1+1/(1-x))$$

or

$$(1+1/(1-x))^{-1/3}$$

- (3) Find all solutions (to 3 decimal places) to the equation  $e^x - x = 5 \sin(x) + 20$ .  
Write down the julia command you use.

$$f(x) = \exp(x) - x - 5\sin(x) - 20$$

$$\text{plot}(f, -20, 20)$$

ok.

$$\text{fzeros}(f, -30, -10)$$

$$-24.1527$$

$$-22.5192$$

$$-19.0423$$

- (4) Write down julia commands to define a function  $f(x)$  which has value  $x^2 - 1$  for  $-1 \leq x \leq 1$  and 0 for other values of  $x$ , and plot its graph to check you are correct.

$$f(x) = \begin{cases} x^2 - 1 & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{plot}(f, -4, 4)$$

(5) Use Julia to find  $\lim_{x \rightarrow 0} \frac{e^{2x^2} - 1}{\sin^2(3x)}$ , by any method.

$$f(x) = (\exp(2x^2) - 1) / \sin(3x)^2$$

$$\text{limit}(f, 0)$$

$$\frac{2}{9}$$

- (6) Consider the function  $f(x) = 20e^{-x^2-4x-4} + 3x$ . Use julia to find all the critical points; write both the julia commands and your answers.

$$f(x) = 20\exp(-x^2-4x-4) + 3x$$

$$\text{plot}([f, D(f)], -20, 20)$$

ek.

$$\text{zeros}(D(f), -10, 10)$$

$$-1.92457$$

$$-0.220507$$

- (7) Consider a function  $f(x)$  for which  $f'(x) = \frac{2}{1+x^2} - 1$ . Use Julia to find all the critical points; write both the Julia commands and your answers. Where is the function concave up and concave down?

$$fp(x) = 2 / (1 + x^2) - 1$$

$$plot(fp, -20, 20)$$

or

$$fzeros(fp, -10, 10)$$

$$-1.0$$

$$+1.0$$

$$plot(D(fp), -20, 20)$$

$$fzeros(D(fp), -10, 10)$$

$$0$$

$(-\infty, 0)$  concave up

$(0, \infty)$  concave down



(8) Use the built in Newton's method `newton(f, fp, x)` to find all zeros of

$$f(x) = \frac{40 \cos(x)}{(e^x - e^{-x})} + 1, \text{ where } fp = D(f).$$

$$f(x) = 40 \cos(x) / (e^x - e^{-x}) + 1$$

$$\text{plot}(f, -10, 10)$$

etc

$$\text{newton}(f, 11(f), -1.5)$$

$$\text{newton}(f, D(f), 1.7)$$

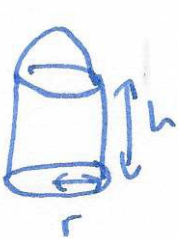
$$\text{newton}(f, D(f), 3.5)$$

$$-1.467875 \dots$$

$$1.70404499 \dots$$

$$3.58699165 \dots$$

- (9) You wish to build a grain silo in the shape of a cylinder with a hemisphere attached on the top. If the total volume should be  $900\text{m}^3$ , what is the smallest surface area possible?



$$V = \pi r^2 h + \frac{1}{2} \frac{4}{3} \pi r^3 = 900$$

$$A = 2\pi r h + \frac{1}{2} 4\pi r^2 + \pi r^2$$

$$A = 2\pi r \left( \frac{900 - \frac{2}{3}\pi r^3}{\pi r^2} \right) + 2\pi r^2 + \pi r^2$$

plot  $(A, 0, 100)$

ch.

fzero  $(D(A), 0, 20)$

$$r = 7.54619$$

$$A(7.54619) = 357.796$$

variant: include base

$$r = 5.56008$$

$$A(5.56008) = 485.60$$

- (10) Use julia to find the area under the curve of  $f(x) = 3e^{\sin(x)}$  between  $x = 2$  and  $x = 4$ . Find the volume of revolution obtained by rotating this region around the  $x$ -axis.

$$f(x) = 3 \exp(\sin(x))$$

$$\text{float}(\text{integrate}(f, 2, 4))$$

$$7.67982\dots$$

$$\text{float}(\text{integrate}(x \rightarrow \pi x f(x)^2, 2, 4))$$

$$114.73825\dots$$