

Math 229 Calculus Computer Lab Spring 16 Final a

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may only use Julia during this exam. No calculators or cell phones or notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Final	
Overall	

- (1) Convert the following julia expressions to standard mathematical expressions. Use parentheses if necessary to clearly indicate the order of operations:

(a) $x+y/2x-z$

$$x + \frac{y}{2x} - z$$

(b) $\sin(3x^2)/x/3$

$$\frac{\sin(3x^2)}{3x}$$

(c) $a-b/(c-a)$

$$a - \frac{b}{c-a}$$

	Final
	Overall

(2) Convert the following standard mathematical expressions into julia expressions.

(a) $\cos^2(\frac{1}{2}x)$

$$\cos(x/2)^2$$

(b) e^{-3x^2}

$$\exp(-3x^2)$$

(c) $\frac{1}{\sqrt[3]{1-\frac{1}{1+x}}}$

$$(1 - 1/(1+x))^{(-1/3)}$$

- (3) Find all solutions (to 3 decimal places) to the equation $e^x + x = 4 \cos(x) - 20$.
Write down the julia command you use.

$$f(x) = \exp(x) + x - 4\cos(x) + 20$$

$$\text{plot}(f, -20, 20)$$

$$\text{etc.} \quad -30, -10$$

$$\text{fzeros}(f, -30, -10)$$

$$-22.79$$

$$-20.5611$$

$$-17.8471$$

- (4) Write down julia commands to define a function $f(x)$ which has value $1-x^2$ for $-1 \leq x \leq 1$ and 0 for other values of x , and plot its graph to check you are correct.

$$f(x) = \begin{cases} 1-x^2 & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$f(x) = \begin{cases} 1-x^2 & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$(0, 1) \text{ interval}$$

$$f(x) = \begin{cases} 1-x^2 & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{plot}(f, -4, 4)$$

(5) Use Julia to find $\lim_{x \rightarrow 0} \frac{e^{3x^2} - 1}{\sin^2(2x)}$, by any method.

$$f(x) = (\exp(3x^2) - 1) / \sin(2x)^2$$

$$\text{limit}(f, 0)$$

$$0 \frac{3}{4} \sin(2x) \rightarrow 1 \Rightarrow x \rightarrow 1 = (x) \frac{3}{4}$$

$$(x, y) \rightarrow (1, 1)$$

- (6) Consider the function $f(x) = 10e^{-x^2-4x-4} - 2x$. Use julia to find all the critical points; write both the julia commands and your answers.

$$f(x) = 10 \exp(-x^2 - 4x - 4) - 2x$$

$$\text{plot}([f, D(f)], -20, 20)$$

etc.

$$\text{zeros}(D(f), -10, 10)$$

$$-3.67963$$

$$-2.10103$$

- (7) Consider a function $f(x)$ for which $f'(x) = \frac{3}{2+x^2} - 1$. Use Julia to find all the critical points; write both the Julia commands and your answers. Where is the function concave up and concave down?

$$f_p(x) = 3 / (2 + x^2) - 1$$

$$\text{plot}(f_p, -20, 20)$$

end

$$\text{fzeros}(f_p, -10, 10)$$

-1.0

+1.0

$$\text{plot}(D(f_p), -20, 20)$$

$$\text{fzeros}(D(f_p), -10, 10)$$

0.0

$(-\infty, 0)$ concave up

$(0, \infty)$ concave down

(8) Use the built in Newton's method `newton(f, fp, x)` to find all zeros of

$$f(x) = \frac{10 \sin(x)}{\left(\frac{1}{10}e^x - e^{-x}\right)} + 1, \text{ where } fp = D(f).$$

$$f(x) = 10 \sin(x) / (\exp(x)/10 - \exp(-x)) + 1$$

`plot(f, -10, 10)`

ek.

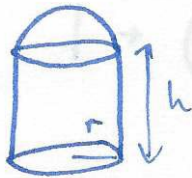
`newton(f, D(f), 0)`

`newton(f, D(f), 3.5)`

0.0814221...

3.4631436...

- (9) You wish to build a grain sill in the shape of a cylinder with a hemisphere attached on the top. If the total volume should be 700m^3 , what is the smallest surface area possible?



$$V = \pi r^2 h + \frac{1}{2} \frac{4}{3} \pi r^3 = 700$$

$$A = 2\pi r h + \frac{1}{2} \frac{4}{3} \pi r^2 + \pi r^2$$

$$A = 2\pi r \left(\frac{700 - \frac{2}{3} \pi r^3}{\pi r^2} \right) + 2\pi r^2 + \pi r^2$$

plot $(A, 0, \infty)$
etc.

$$\text{fzeros}(D(x), 0, \infty) \quad r = 6.93979$$

$$A(6.93979) = 302.60$$

variant: include base

$$r = 5.11328$$

$$A(5.11328) = 410.675$$

- (10) Use `julia` to find the area under the curve of $f(x) = 2e^{\sin(x)}$ between $x = 1$ and $x = 3$. Find the volume of revolution obtained by rotating this region around the x -axis.

$$f(x) = 2\exp(\sin(x))$$

$$\text{float}(\text{integrate}(f, 1, 3))$$

$$8.8495998\dots$$

$$\text{float}(\text{integrate}(x \rightarrow \pi x f(x)^2, 1, 3))$$

$$129.02452\dots$$