Math 229 Calculus Computer Lab Spring 15 Final b

Name: _____Solutions_____

- I will count your best 6 of the following 8 questions.
- You may only use `julia` during this exam. No calculators or cell phones.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
	60	

Final	
Overall	

(1) Convert the following julia expressions to standard mathematical expressions. Use parentheses to clearly indicate the order of operations:

(a) a+y/z-x

$$a + \frac{y}{z} - x$$

(b) sin(1/2x^2)*1/2*x^3

$$\sin\left(\frac{1}{2x^2}\right) \frac{x^3}{2}$$

(c) (a+x)/c*2+a/x*3

$$\frac{2(a+x)}{c} + \frac{3a}{x}$$

(2) Find all solutions (to at least 3 decimal places) to the equation $12\sin(2x) = 5x - 100$. Write down the julia command you use.

$$f(x) = 12\sin(2x) - 5x + 100$$

$$\texttt{plot}(f, 15, 25)$$

$$\texttt{fzero}(f, 17) = 17.8390830197961$$

$$\texttt{fzero}(f, 19) = 18.516334438670864$$

$$\texttt{fzero}(f, 20) = 20.347666072632563$$

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(3) Use `julia` to find $\lim\limits_{x\to 0} \dfrac{\cos(5x)-1}{e^{4x^2}-1}$, by any method.

$xs = [1/10^{\wedge}i \text{ for } i \text{ in } 1:10]$

$f(x) = (\cos(5x)-1)/(e^{4x^2}-1)$

$\text{map}(f, xs)$

-2.9964
-3.12372
-3.12499
-3.125
-3.125
-3.12507
-3.12778
-2.75
NaN
NaN

$\boxed{-3.125}$

(4) Consider the function $f(x) = e^{-x} + e^x - 200x^2$. Use julia to find all the critical points; write both the julia commands and your answers.

$$f(x) = \exp(-x) + \exp(x) - 200x^2$$

$$\text{plot}(f, -10, 10)$$

$$\text{fzero}(D(f), -7) = -9.080977483743737$$

$$\text{fzero}(D(f), 0) = 0$$

$$\text{fzero}(D(f), 7) = 9.080977483743737$$

(5) Consider a function $f(x)$ for which $f'(x) = 2\cos(x) + x^2 - 10$. Use julia to find all the critical points and where is the function concave up and concave down; write both the julia commands and your answers.

$fp(x) = 2\cos(x) + x^2 - 10$

$plot(fp, -10, 10)$

$fzero(fp, -3) = -3.4504177505947657$

$fzero(fp, 3) = 3.4504177505947657$

$plot(D(fp), -10, 10)$

$fzero(D(fp), 0) = 0$

concave up $(0, \infty)$

concave down $(-\infty, 0)$

(6) Use the built in Newton's method `newton(f, fp, x)` to find all zeros of $f(x) = \dfrac{30\sin(x)}{(x^2+1)} + 1$, where `fp(x) = D(f)(x)`; write both the `julia` commands and your answers.

$$f(x) = 30\sin(x)/(x^2+1) + 1$$

$$\text{plot}\left(f, -10, 10\right)$$

$$\text{newton}\left(f, D(f), -3\right) = -2.835499893548154$$

$$0 = -0.033376663319676657$$

$$3 = 3.635226532803391$$

$$5 = 5.134663692498015$$

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(7) You wish to construct a tin can which is a cylinder with a base but no top, which has total surface area 5m^2 and maximal volume. What are the dimensions of the tin can? Write both the `julia` commands and your answers.

[The volume of a cylinder of height h and radius r is $V = \pi r^2 h$. The area of a disc of radius r is πr^2, the circumference of a disc is $2\pi r$]

$$V = \pi r^2 h$$

$$A = \pi r^2 + 2\pi r h = 5$$

$$h = \frac{5 - \pi r^2}{2\pi r}$$

$$V = \pi r^2 \frac{(5 - \pi r^2)}{2\pi r} = \frac{r}{2}(5 - \pi r^2)$$

$$f(x) = \frac{x}{2}(5 - \pi \ast x^{\wedge}2)$$

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plot(f,0,2)
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fzero(D(f), 0.7) = 0.7283656203947194 = r
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$$\Rightarrow \quad h = \frac{5 - \pi r^2}{2\pi r} = 0.7283656203947193 = h$$

(8) Use `julia` to find the area under the curve of $f(x) = \sin^2(x^2)$ between 1 and 8. Write both the `julia` commands and your answers.

$$f(x) = \sin(x^2)^2$$

$$\text{quadgk}(f, 1, 8)$$

$$= 3.6009337126007543$$