Math 229 Calculus Computer Lab Spring 15 Final a

Name: Solutions

- I will count your best 6 of the following 8 questions.
- You may only use `julia` during this exam. No calculators or cell phones.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
	60	

Final	
Overall	

(1) Convert the following `julia` expressions to standard mathematical expressions. Use parentheses to clearly indicate the order of operations:

(a) `a-y/z+x`

$$a - \frac{y}{z} + x$$

(b) `cos(1/3x^2)*1/3*x^3`

$$\cos\left(\frac{1}{3x^2}\right)\frac{x^3}{3}$$

(c) `(a+x)/c*3+a/y*5`

$$3\frac{(a+x)}{c} + \frac{5a}{y}$$

(2) Find all solutions (to at least 3 decimal places) to the equation $12\cos(2x) = -5x - 100$. Write down the `julia` command you use.

```
f(x) = 12cos(2x) + 5x + 100
plot(f, -25, -15)
fzero(f, -23) = -22.197840389848597
fzero(f, -21) = -21.559369921862373
fzero(f, -20) = -19.698031391109765
```

(3) Use `julia` to find $\lim_{x\to 0} \frac{\cos(7x)-1}{e^{3x^2}-1}$, by any method.

```
xs = [1/10^i for i in 1:10]
f(x) = (cos(7x)-1)/(exp(3x^2)-1)
map(f,xs)
```

```
-7.7246
-8.16211
-8.16662
-8.16667
-8.16667
-8.16653
-8.17407
-11.0
NaN
NaN
```

-8.16667

(4) Consider the function $f(x) = e^{-x} + e^{x} - 100x^2$. Use `julia` to find all the critical points; write both the `julia` commands and your answers.

```
f(x) = exp(-x) + exp(x) - 100x^2
plot(f, -10, 10)
fzero(D(f), -7) = -7.283997681276688
fzero(D(f), 0) = 0
fzero(D(f), 7) = 7.283997681276688
```

(5) Consider a function $f(x)$ for which $f'(x) = 2\sin(x) + x^2 - 2$. Use `julia` to find all the critical points and where is the function concave up and concave down; write both the `julia` commands and your answers.

```
fp(x) = 2sin(x)+x^2 - 2
plot(fp,-10,10)
fzero(fp,-2) =  -1.9618842464108348
fzero(fp,1) =  0.7749808144230433
plot(D(fp),-10,10)
fzero(D(fp),0) =  -0.7390851332151607
```

concave up $(-0.7, \infty)$

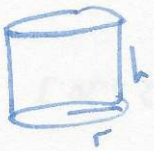
concave down $(-\infty, -0.7\ldots)$.

(6) Use the built in Newton's method `newton(f, fp, x)` to find all zeros of $f(x) = \frac{15\cos(x)}{(x^2+1)} + 1$, where `fp(x) = D(f)(x)`; write both the `julia` commands and your answers.

```
f(x) = (15cos(x))/(x^2+1) + 1
plot(f,-10,10)
newton(f,D(f),-3) =   -3.562452655213230 3
                -2    -1.877143426891468 2
                 2     1.877143426891468 2
                 3     3.562452655213230 3
```

(7) You wish to construct a tin can which is a cylinder with a base but no top, which has total surface area 3m^2 and maximal volume. What are the dimensions of the tin can? Write both the `julia` commands and your answers.

[The volume of a cylinder of height h and radius r is $V = \pi r^2 h$. The area of a disc of radius r is πr^2, the circumference of a disc is $2\pi r$]



$V = \pi r^2 h$

$A = \pi r^2 + 2\pi r h = 3$

$h = \frac{3 - \pi r^2}{2\pi r}$

$V = \frac{\pi r^2 (3 - \pi r^2)}{2\pi r} = \frac{r}{2}(3 - \pi r^2)$

$f(x) = x/2 * (3 - \pi * x^2/2)$

$plot(f, 0, 2)$

$fzero(D(f), 0.5) = 0.5641895835477563 = r$

$\Rightarrow h = \frac{3 - \pi r^2}{2\pi r} = 0.5641895835477563. = h$

(8) Use `julia` to find the area under the curve of $f(x) = \sin^2(x^2)$ between 1 and 5. Write both the `julia` commands and your answers.

f(x) = sin(x^2)^2
quadgk(f, 1, 5)

= 2.1190399849548293