CSI, Dept of Math, Franzblau, F 2011

Calculus II (Math 232), Final Exam

Name: ____

Check that you have **9 pages**, including this one, with 10 problems (100 pts total).

DO ONLY ONE OF PROBLEMS 10A and 10B.

Circle the label of the problem you want graded, otherwise I'll grade the first problem attempted.

You may use the handout on series convergence tests (return with exam).

Any type of calculator is allowed, except cell-phones. Sharing of calculators, erasers, etc is not allowed. Texts or notes are not allowed. Scratch paper must be obtained from the instructor and returned with the exam.

For full credit, **you must show work and/or give reasons**, unless stated otherwise.

You are expected to do your own work and not discuss the test with anyone except the instructor. Copying or collaboration is grounds for a grade of zero. There are multiple versions of this test.

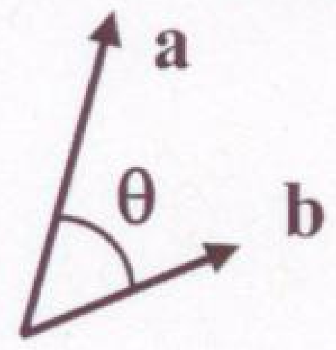
Vector Facts and Formulas.

$\mathbf{i} =< 1, 0, 0 >$, $\mathbf{j} =< 0, 1, 0 >$, and $\mathbf{k} =< 0, 0, 1 >$ are the standard unit vectors.

Dot product in component form: $< a, b, c > \cdot < d, e, f >= ad + be + cf$.

Dot product in geometric form: $\mathbf{a} \cdot \mathbf{b} = ||\mathbf{a}||\ ||\mathbf{b}|| \cos\theta$

where θ is the angle between vectors $\mathbf{a}$ and $\mathbf{b}$ and $||\mathbf{v}||$ represents the length of vector $\mathbf{v}$.



Cross product in component form. $< a, b, c > \times < d, e, f >=$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a & b & c \\ d & e & f \end{vmatrix} = \mathbf{i}\begin{vmatrix} b & c \\ e & f \end{vmatrix} - \mathbf{j}\begin{vmatrix} a & c \\ d & f \end{vmatrix} + \mathbf{k}\begin{vmatrix} a & b \\ d & e \end{vmatrix} \quad \text{where} \quad \begin{vmatrix} u & v \\ w & x \end{vmatrix} = ux - wv.$$

Cross product distributive property: $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) + (\mathbf{a} \times \mathbf{c})$. $(\mathbf{b} + \mathbf{c}) \times \mathbf{a} = (\mathbf{b} \times \mathbf{a}) + (\mathbf{c} \times \mathbf{a})$.

Length of the cross product is the area of the parallelogram spanned by the two vectors: $||\mathbf{a} \times \mathbf{b}|| = ||\mathbf{a}||\ ||\mathbf{b}|| \sin\theta$.

1. (*15 pts*) Let R be the region bounded by $y = \sqrt{x}$ and $y = x^2$ (same as $x = y^2$ and $x = \sqrt{y}$). The intersection points are $(0,0)$ and $(1,1)$. See graph below.

Rotate R about the **y-axis** to obtain a solid, S.

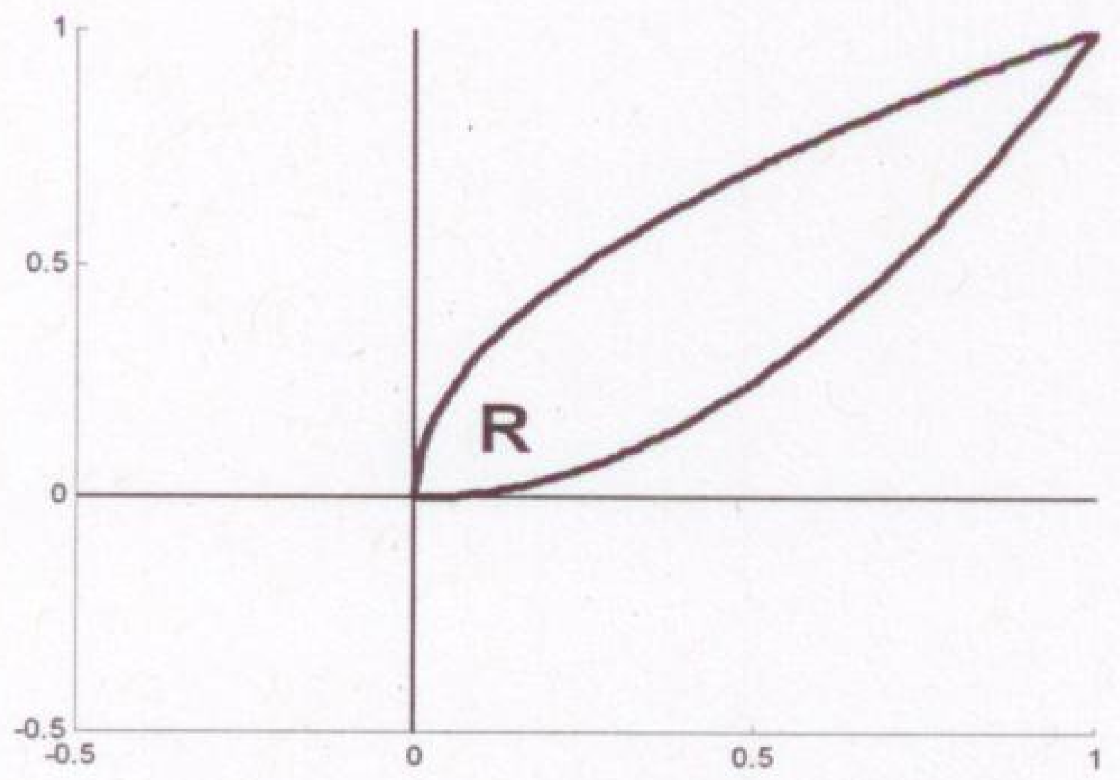
a. (1 pts) Sketch S (looks like a bowl with a trumpet horn removed).

b. (7 pts) Set up an integral for the volume of S using the **washer method**.

c. (7 pts) Set up an integral for the volume of S using the **cylinder method** (shell method).

• In each case, explain the geometric meaning of each term in the integral (radius, thickness, etc) using words and pictures.

• You do **not** have to evaluate the integrals. (However, if you have time at the end of the test, you may wish to do this to check your answers.)



2. (*10 pts*) The improper integral $A = \int_1^\infty \frac{1}{x^k}\,dx$ represents the area of an infinite region.

a. (8 pts) Give an example of a positive value of k such that the area is **finite**.
Compute the area by evaluating the integral
(use limits!—substitution of ∞ is not allowed).

b. (2 pts) Sketch the graph of $\frac{1}{x^k}$ roughly, and shade in the region (the visible part).

3. (*10 pts*)

a. (4 pts) For each of the integrals below, write down the most appropriate method of integration: SB (substitution), IBP (integration by parts), or PF (partial fractions).

You do **not** have to evaluate the integrals.

- $\int xe^{-x^2}\,dx$ method: ____________
- $\int xe^{-x/2}\,dx$ method: ____________
- $\int \frac{3}{x^2-4}\,dx$ method: ____________
- $\int \frac{3x}{x^2+4}\,dx$ method: ____________

b. (6 pts) Evaluate the integral $\int xe^{-x/2}\,dx$. Show your calculations step by step (please organize clearly for the reader!)

4. (*20 pts*) Let S be the infinite series $S = \sum_{m=1}^{\infty} s_m = \sum_{m=1}^{\infty} \frac{\sqrt{m}}{m^2+3}$

Determine whether S converges or diverges using at least one of the following tests. Circle the test(s) used.

Limit Comparison Direct Comparison p-Series Geometric Series

Show your reasoning and calculations step by step, and give your conclusion.
If you use the dominant term theorem for limits (DTT), show precisely where it is used. If you use a table for limits, it must show a clear trend.
If you use the p-Series Test, give the value of p. If you use the Geometric Series Test, give the value of r.

5. (*10 pts*) Let $P(x)$ be the power series $P(x) = \sum_{n=0}^{\infty} n \cdot 3^n \cdot x^n$

Find the radius of convergence R. Justify your answer.

Reminder: the series must converge for $|x| < R$ and diverge for $|x| > R$.

Show all calculations step by step. If you use the dominant term theorem for limits (DTT), show precisely where it is used. If you use a table for limits, it must show a clear trend.

6. (*10 pts*) *Give an example* of a series $\sum_{k=1}^{\infty} b_k$ with positive terms $(b_k > 0)$, such that $\sum_{k=1}^{\infty} b_k$ and $\sum_{k=1}^{\infty} (-1)^{k-1} b_k$ both converge.

• For each series, write down a test that will prove convergence and explain briefly (full details not needed).

7. (*5 pts*) Sketch the vectors described below. If it is not clear from the drawing, describe your procedure briefly in words.

a. (2 pts) Sketch the vector $-\mathbf{u}$.

b. (3 pts) Sketch the vector $\mathbf{u} + \mathbf{v}$.

8. (*5 pts*) Compute the cross product $3\,\mathbf{j} \times (\mathbf{i} - 4\,\mathbf{k})$, and write in the form $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$. Show calculations step by step (formulas on front page, if needed).

9. (*5 pts*) Determine the angle between the vectors $\mathbf{a} =< -1, 0, 2 >$ and $\mathbf{b} =< 3, 0, -6 >$ (in radians). You may use any method (see front page). Show any calculations step by step. *Check*: the answer should be simple.

10. (*10 pts*) Do ONE of problem A (all parts) or problem B. Circle the letter of the problem you want graded.

- **A**. (*Eqn of a line.*) Let P be the point $(-3, 2)$ and let $\mathbf{v}$ be the vector $< 1, -2 >$. Let L be the line through P and parallel to $\mathbf{v}$.

  • Sketch the line, and give the coordinates of at least one other point on the line.

  • Give the vector equation of the line in the form $\mathbf{L}(t) =< a + ct, b + dt >$.
  (a, b, c, d are constants, and t is the variable parameter.)

  Show the derivation step by step. *Reminder*: Use vector $\overrightarrow{OP}$ where $O = (0, 0)$.

- **B**. (*Eqn of a plane.*) Let Q be the point $(0, -4, 1)$ and let $\mathbf{n}$ be the vector $< -3, 1, 2, >$. Let P be the plane through Q, normal ($\perp$) to $\mathbf{n}$.

  Give the equation of the plane in the form $ax + by + cz = d$, where a, b, c, d are constants, and x, y, z are the standard 3D variables.

  Reminder: Every vector $\overrightarrow{QX}$, where $X = (x, y, z)$ must be perpendicular ($\perp$) to $\mathbf{n}$.