Math 232. Fall 2011. Final.
Answer all questions. Show all work.

Please explain your reasoning in complete sentences. Answers submitted without justification will not receive full credit.

1) Evaluate $\int x^2 \ln(x)\, dx$.

2) Find the volume of the solid of revolution formed by revolving the region bounded by $y = 8x$ and $y = x^4$ around the y-axis.

3) Find the equation of the plane which contains $(3, 1, 2), (2, 4, 1)$ and $(4, -1, 0)$.

4) Evaluate $\int \sin^2(x) \cos^5(x)\, dx$.

5) Find the parametric representation for the line which contains $(1, 3, 1)$ and $(2, 2, 5)$.

6) Determine if the series

$$\sum_{n=2}^{\infty} \frac{3n+4}{n^3-n}$$

converge or diverge? Explain your answer.

7) Find the interval of convergence for

$$\sum_{n=1}^{\infty} \frac{2^n (x+3)^n}{n}.$$

8) Evaluate

$$\int \frac{x}{(x+3)^4}\, dx.$$

9) Find the Taylor series for $(1+x)^{1/5}$ centered at $c = 0$. Use the first 3 terms to approximate $(1.1)^{1/5}$, and use Taylor's theorem to estimate the error.

10) Calculate $\int_1^{\infty} \frac{1}{x^7}\, dx$. Show all work.

PLEASE TURN OVER

Some useful expressions:

$$\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}, \qquad \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\,\mathbf{v}$$

$$(u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1)$$

$$\sum f^{(n)}(c)\frac{(x-c)^n}{n!}, \qquad \binom{a}{n} = \frac{a(a-1)\cdots(a-(n-1))}{n!}$$

$$\pi\int_a^b (R(x))^2 - (r(x))^2\,dx, \qquad 2\pi\int_a^b r(x)h(x)\,dx,$$

$$(\tan(x))' = \sec^2(x), \qquad (\sec(x))' = \tan(x)\sec(x)$$

$$\sin^2(x) = \frac{1-\cos(2x)}{2}, \qquad \cos^2(x) = \frac{1+\cos(2x)}{2}, \qquad \sec^2(x) = 1+\tan^2(x)$$

$$\int \tan(x)\,dx = \ln|\sec(x)|, \qquad \int \sec(x)\,dx = \ln|\sec(x)+\tan(x)|$$

$$\int \sin^n x\,dx = -\frac{\sin^{n-1}x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2}x\,dx$$

$$\int \cos^n x\,dx = \frac{\cos^{n-1}x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2}x\,dx$$

$$\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx$$

$$\int \sec^n x\,dx = -\frac{\sec^{n-2}x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx$$