CSI, Dept of Math, Franzblau, Sp 10

Calculus II (Math 232), Final Exam

Name: _____

Check that you have **10 pages**, including this one, with 9 problems.

DO ONLY ONE OF PROBLEMS 5, 6.

Circle the problem you want graded, otherwise I'll grade the first problem attempted.

You should have a handout on series convergence tests (return with exam).

Any type of calculator is allowed, except cell-phones. Sharing of calculators, erasers, etc is not allowed. Texts or notes are not allowed. Scratch paper must be obtained from the instructor and returned with the exam.

For full credit, **you must show work and/or give reasons**, unless stated otherwise.

You are expected to do your own work and not discuss the test with anyone except the instructor. Copying or collaboration is grounds for a grade of zero. There are multiple versions of this test.

The following formulas are for reference: you may or may not need them for test problems.

Trig Identities:

- $\cos^2\theta + \sin^2\theta = 1, \quad \tan^2\theta + 1 = \sec^2\theta$
- $\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta, \quad \sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$

Trig Derivatives/Integrals:

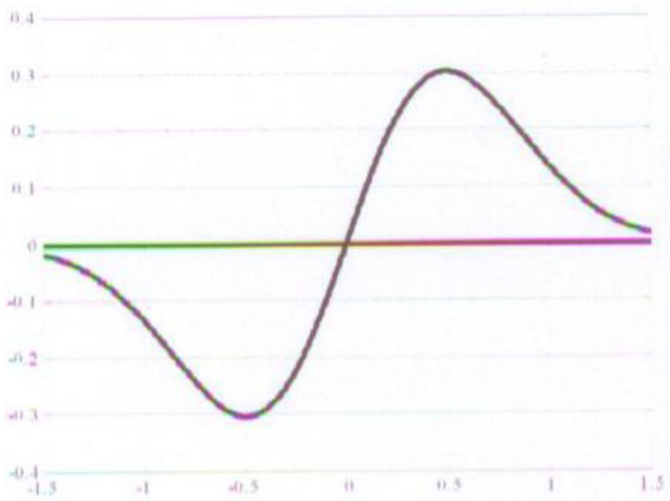
- $\frac{d}{dx}\tan x = \sec^2 x \qquad \int \sec^2 x\ dx = \tan x + C$
- $\frac{d}{dx}\cot x = -\csc^2 x \qquad \int \csc^2 x\ dx = -\cot\ x + C$
- $\frac{d}{dx}\sec x = \sec x \tan x \qquad \int \sec x \tan x\ dx = \sec x + C$
- $\frac{d}{dx}\csc x = -\csc x \cot x \qquad \int \csc x \cot x\ dx = -\csc x + C$

1. (*10 pts*) Let R be the infinite region bounded by the x-axis and $y = xe^{-2x^2}$, on the interval $0 \leq x < \infty$.

(a) Shade the visible part of the region on the graph below.

(b) Write down an (improper) integral. for the area of R, and write down the *definition* of your integral as a limit. *Use* this definition in Part (c).

(c) Find the area of R if it is finite, or show that the integral diverges.

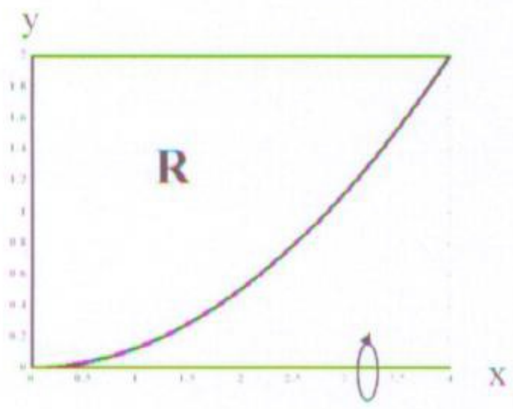


2. (*15 pts*) Let R be the region bounded by $y = \dfrac{x^2}{8}$ and the y-axis, for $0 \leq y \leq 2$, as drawn below. Rotate R about the **x**-axis to obtain a solid, S.

a. (2 pts) Sketch S (it should look like a cylinder with a curved cone removed).

b. (7 pts) Set up an integral for the volume of S, and explain the meaning of the terms in the integral (using words like area, height, radius, etc).
Reminder: you may need to rewrite $y = \frac{x^2}{8}$ as $x = g(y)$.
Method used (circle one): Disk/Washer Cylindrical Shell

c. (6 pts) Find the volume of S by evaluating the integral. Show your calculations step by step. *Check*: Your answer should be a number of the form $c\pi$ where $10 \leq c \leq 15$.



3. (*20 pts*) Find the integrals below. State the methods used (algebra, substitution, integration by parts, or trig substitution) and show details of the calculation step by step. **Label clearly which work matches which problem**.

a. (7 pts) $\displaystyle\int \frac{3}{y^2+5}\,dy$

b. (7 pts) $\displaystyle\int t\sin(3t)\,dt$

c. (6 pts) $\displaystyle\int \frac{1}{x(\ln x)^3}\,dx$

4. (*10 pts*) Let $G = \sum_{n=0}^{\infty} \frac{7(-1)^n 3^n}{4^n}$.

a. (5 pts) Show that G is a geometric series (put it in the standard form $c\sum r^n$) and determine whether the series converges. Justify your answer.

b. (5 pts) If G converges, find the exact sum, and justify your answer.

If G diverges, determine its behavior, and justify your answer. *Circle one*:

Diverges to $+\infty$ Diverges to $-\infty$ Sum does not exist

DO ONLY ONE OF PROBLEMS 5, 6. Circle the problem you want graded.

5. (*15 pts*) Let A be the series $\sum \dfrac{m^3 + 3m^2}{\sqrt{m} + 2m^4}$

Determine whether A converges using the *Limit Comparison Test*. Show the details of your calculation step by step, and justify your conclusion. *Conclusion* (circle one):

Converges Diverges to $+\infty$ Diverges to $-\infty$ Sum does not exist

Reminder: You must create a second series B to compare to A, and determine whether B converges using an appropriate test.

DO ONLY ONE OF PROBLEMS 5, 6. Circle the problem you want graded.

6. (*15 pts*) Let S be the series $\sum \dfrac{(-1)^n\ 7n}{3^n}$.

Determine whether S converges using the *Ratio Test*. Show the details of your calculation step by step and justify your conclusion.

Warning to creative algebraists: $3^n \neq 3n$ and $7n \neq 7^n$.

Conclusion (circle one):

Converges Diverges to $+\infty$ Diverges to $-\infty$ Sum does not exist

7. (*10 pts*) *Give an example* of a series $\sum c_m$ where the terms are *positive* ($c_m > 0$) and *decreasing* ($c_m > c_{m+1}$), but $\sum c_m$ diverges. *Suggestion*: p-series.

- Demonstrate that the terms are decreasing using rules of inequalities, an appropriate graph, or an appropriate derivative. (Writing out a few terms is not enough.)
- Explain why the series diverges using an appropriate convergence test.

8. (*10 pts*)

a. (5 pts) Sketch the vector $\mathbf{u} - 2\mathbf{w}$. Explain your answer using words and/or pictures.

b. (5 pts) Sketch and describe the line represented by the function $L(t) = \mathbf{u} + t\mathbf{w}$, where t is a scalar that ranges over the set of real numbers.
Assume the tail of $\mathbf{u}$ is at the origin.

9. (*10 pts*) Let **a** be the vector $< 2, 0, 5 >$ and **b** the vector $< 0, -1, 1 >$.

a. (2 pts) Compute the lengths $||\mathbf{a}||$ and $||\mathbf{b}||$ of vectors **a** and **b**. Show the calculations step by step.

b. (3 pts) Find a unit vector (length 1) in the direction of vector **a**. Write your answer in the form $< a, b, c >$. Justify your answer.

c. (5 pts) Let θ be the angle between **a** and **b** (see sketch). Recall that the dot product is defined in two equivalent ways:
$\mathbf{a} \cdot \mathbf{b} = ||\mathbf{a}||\ ||\mathbf{b}|| \cos\theta$ and
$< a, b, c > \cdot < d, e, f >= ad + be + cf$.

Using the dot product, find the angle between **a** and **b** (in radians). Show your calculations step by step.

Check: your answer should lie between $\pi/6$ and $\pi/3$.
Label clearly which work matches which problem.