double angle formulae

$$e^{i2\theta} = \cancel{e}\ \cos 2\theta + i\sin 2\theta$$

$$\| $$

$$e^{i\theta}\cdot e^{i\theta} = (\cos\theta + i\sin\theta)^2 = \cos^2\theta + 2i\sin\theta\cos\theta - \cancel{\cos}\sin^2\theta .$$

$$\cancel{\cos\theta}\cdot \cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta .$$

triple angle formulae

$$e^{i3\theta} = \cos 3\theta + i\sin 3\theta$$

$$\|$$

$$(e^{i\theta})^3 = (\cos\theta + i\sin\theta)^3 = \cos^3\theta + 3\cos^2\theta\, i\sin\theta + 3\cos\theta(i\sin\theta)^2 + (i\sin\theta)^3.$$

$$= \cos^3\theta + i\,3\cos^2\sin\theta - 3\cos\theta\sin\theta - i\sin^3\theta$$

$$\cos 3\theta = \cos^3\theta + 3\cos\theta\sin^2\theta = \cos\theta(1+3\sin^2\theta)$$

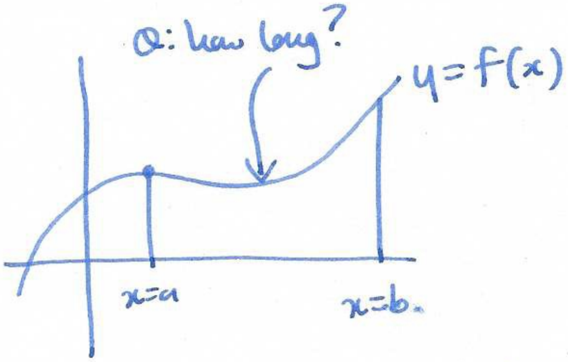
$$\sin 3\theta = 3\cos^2\sin\theta - \sin^3\theta = \sin\theta(3\cos^2\theta - 1) .$$

How to write l'Hôpital's rule problems

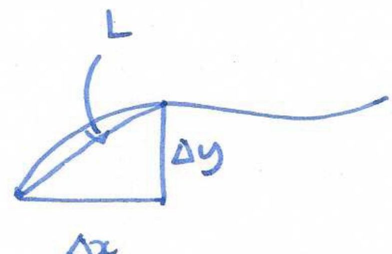
$$\left.\begin{aligned} \sin(2x) &\simeq 2x - \frac{(2x)^3}{3!} + O(x^5). \\ e^{3x} &\simeq 1 + 3x + O(x^2). \end{aligned}\right\}.$$

$$\frac{\sin(2x)}{e^{3x}-1} \simeq \frac{2x}{3x} = \frac{2}{3} .$$

§8.1 Arc length and surface area



A:

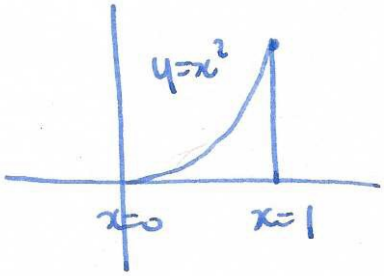


$$|L| \simeq \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$\text{length} = \sum_{i=1}^{n} |L_i| = \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} = \sum_{i=1}^{n} \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2}\,\Delta x_i$$

take limit as $|\Delta x_i| \to 0$: arc length $= \int_a^b \sqrt{1+(f'(x))^2}\,dx = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

Example



$$\frac{dy}{dx} = 2x \qquad \text{arc length} = \int_0^1 \sqrt{1+4x^2}\,dx$$

(trig sub integral).

Example $f(x) = \frac{1}{12}x^3 + \frac{1}{x}$ on $[1,2]$

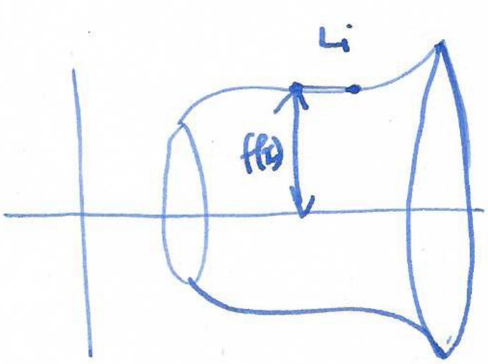
$$f'(x) = \frac{1}{4}x^2 - \frac{1}{x^2}.$$

$$\int_1^2 \sqrt{1+\left(\tfrac{1}{4}x^2 - \tfrac{1}{x^2}\right)^2}\,dx = \int_1^2 \sqrt{1+\frac{1}{16}x^4 - \frac{1}{2} + \frac{1}{x^4}}\,dx$$

$$= \int_1^2 \sqrt{\frac{1}{16}x^4 + \frac{1}{2} + \frac{1}{x^4}}\,dx = \int_1^2 \sqrt{\left(\frac{x^2}{4} + \frac{1}{x^2}\right)^2}\,dx = \int_1^2 \frac{x^2}{4} + \frac{1}{x^2}\,dx$$

$$= \left[\frac{x^3}{12} - \frac{1}{x}\right]_1^2 = \frac{8}{12} - \frac{1}{4} - \frac{1}{12} + 1 = 1\frac{1}{3}.$$

Area of surface of revolution

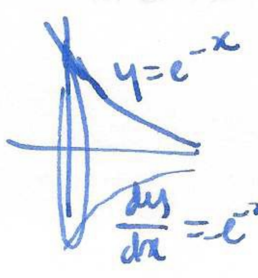


$$\text{area} \simeq \sum |L_i|\, f(x)\, 2\pi.$$

take limit as $|\Delta x_i| \to 0$

$$\text{surface area of vol. of revolution} = \int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$$

Example



$$\text{surface area} \int_0^\infty 2\pi e^{-x}\sqrt{1+(-e^{-x})^2}\,dx$$

sub $u=e^{-x}$

etc ...