so $$\boxed{a_n = \frac{f^{(n)}(c)}{n!}}$$

<u>Examples</u> $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$

$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$

<u>Thm</u> If $f(x)$ is represented by a power series centered at $x=c$, with radius of convergence $R>0$, then $f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$, where $a_n = \frac{f^{(n)}(c)}{n!}$

<u>Useful facts</u> If the Taylor series are convergent, then we can

- differentiate
- integrate
- multiply (xe^x)
- substitute (e^{x^2})

<u>Examples</u> xe^x, e^{-x^2}, $e^x \sin(x)$.

<u>Fact</u> works for complex numbers $x+iy$

$\cancel{e^{x+iy} = e^x \cdot e^{iy}}$ $e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \cdots$

$= 1 + i\theta - \frac{\theta^2}{2!} - \frac{i\theta^3}{3!} + \frac{\theta^4}{4!} = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots$

$+ i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right)$.

$= \cos\theta + i\sin\theta$ $\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$