$$\lim_{R\to\infty} e^{-R}(R-1) = \lim_{R\to\infty} \frac{-R-1}{e^R} \underset{l'H:}{=} \lim_{R\to\infty} \frac{-1}{e^R} = 0.$$

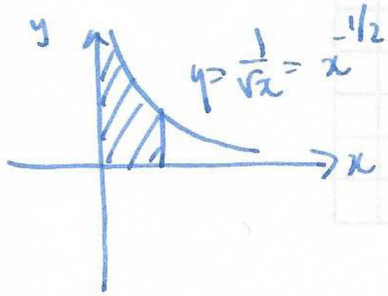
so $\int_0^\infty x e^{-x}\,dx = 1$

<u>Example</u> when does $\int_1^\infty \frac{1}{x^p}\,dx$ converge $\left(\begin{matrix} p=1 & \text{no} \\ p=2 & \text{yes}\end{matrix}\right)$

$$\lim_{R\to\infty} \int_1^R x^{-p}\,dx = \lim_{R\to\infty} \left[\frac{x^{-p+1}}{-p+1}\right]_1^R = \lim_{R\to\infty} \left(\frac{R^{-p+1}}{-p+1} - \frac{1}{-p+1}\right)$$

$$\lim_{R\to\infty} R^{-p+1} = 0 \text{ if } p>1$$
$$= \infty \text{ if } p<1 \quad (\text{in fact } p \le 1 \text{ by } \ln|R| \to \infty).$$

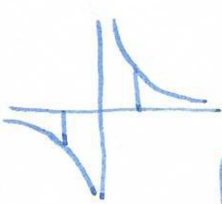
<u>Intervals containing discontinuities</u>



$$\int_0^1 x^{-1/2}\,dx = \lim_{R\to 0} \int_R^1 x^{-1/2}\,dx$$

$$= \lim_{R\to 0} \left[2x^{1/2}\right]_R^1 = \lim_{R\to 0} 2 - 2\sqrt{R} = 2.$$

<u>Warning</u> $\underset{\text{DNE!}}{\int_{-1}^1 \frac{1}{x}\,dx} = \left[\ln|x|\right]_{-1}^1 = \ln|1| - \ln|-1| = 0.$ WRONG!



interval $[-1,1]$ contains a discontinuity!

$$\int_{-1}^1 \frac{1}{x}\,dx = \int_{-1}^0 \frac{1}{x}\,dx + \int_0^1 \frac{1}{x}\,dx$$

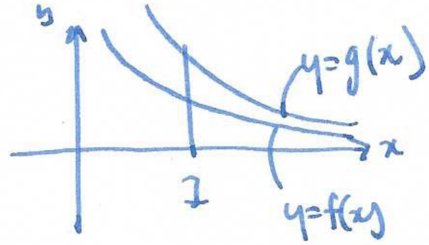
$$= \lim_{R\to 0} \int_{-1}^{-R} \frac{1}{x}\,dx + \lim_{R\to 0} \int_R^1 \frac{1}{x}\,dx$$

$$= \lim_{R\to 0} \left[\ln|x|\right]_{-1}^{-R} + \lim_{R\to 0} \left[\ln|x|\right]_R^1$$

$$= \lim_{R\to 0} \ln|R| + \lim_{R\to 0} -\ln|R| = \underset{\text{DNE.}}{\infty - \infty}$$

comparison test

$$0 \le f(x) \le g(x) \quad \text{on } [1,\infty)$$



if $\int_1^\infty g(x)\,dx$ converges then $\int_1^\infty f(x)\,dx$ converges.

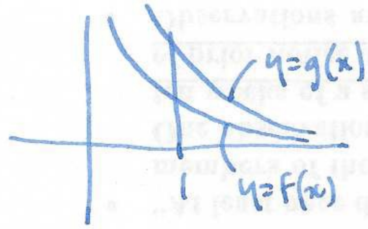
Example $\displaystyle\int_1^\infty \frac{1}{\sqrt{x^3+1}}\,dx$ note: $\sqrt{x^3+1} \ge \sqrt{x^3}$

so $\displaystyle\frac{1}{\sqrt{x^3+1}} \le \frac{1}{\sqrt{x^3}}$

try: $\displaystyle\int_1^\infty \frac{1}{\sqrt{x^3}}\,dx = \int_1^\infty x^{-3/2}\,dx = \lim_{R\to\infty}\left[-2x^{-1/2}\right]_1^R = \lim_{R\to\infty} \frac{-2}{\sqrt{R}} + 2 = 2 < \infty$ converges

so $\displaystyle\int_1^\infty \frac{1}{\sqrt{x^3}}\,dx$ converges $\Rightarrow \displaystyle\int_1^\infty \frac{1}{\sqrt{x^3+1}}\,dx$ converges (but don't know exact value)

other way



$$0 \le f(x) \le g(x) \quad \text{on } [1,\infty)$$

if $\int_1^\infty f(x)\,dx$ diverges $\Rightarrow \int_1^\infty g(x)\,dx$ diverges.

note $\displaystyle\int_1^\infty g(x)\,dx$ diverges $\not\Rightarrow \displaystyle\int_1^\infty f(x)\,dx$ diverges

$\displaystyle\int_1^\infty f(x)\,dx$ converges $\not\Rightarrow \displaystyle\int_1^\infty g(x)\,dx$ converges.

Example

show $\displaystyle\int_2^\infty \frac{1}{x-\sqrt{x}}\,dx$ diverges.

$$x - \sqrt{x} < x \quad (\text{for } x > 1)$$

$$\frac{1}{x-\sqrt{x}} > \frac{1}{x}$$

$$\int_2^\infty \frac{1}{x}\,dx = \lim_{R\to\infty}\int_1^R \frac{1}{x}\,dx = \lim_{R\to\infty}\left[\ln|x|\right]_1^R = \lim_{R\to\infty} \ln|R| - 0 \to \infty$$

$\Rightarrow \displaystyle\int_2^\infty \frac{1}{x-\sqrt{x}}\,dx$ diverges