$$\int e^x \sin x\,dx = -e^x\cos x + e^x \sin x - \int e^x \sin x\,dx$$

$$2\int e^x \sin x\,dx = e^x(\sin x - \cos x) + c$$

$$\int e^x \sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x) + c \qquad \text{check!}$$

§7.2 Trig integrals

$$\int \sin^m x \cos^n x\,dx\ ?$$

techniques:
- $\cos^2 x + \sin^2 x = 1$
- $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$
- sub $u = \sin x \qquad \frac{du}{dx} = \cos x$
- sub $u = \cos x \qquad \frac{du}{dx} = -\sin x$
- parts

Examples

$$\int \sin^2 x\,dx = \int \frac{1}{2} - \frac{1}{2}\cos 2x\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$$

check!

$$\int \sin^3 x\,dx = \int (1-\cos^2 x)\sin x\,dx \qquad \text{sub } u = \cos x \quad \frac{du}{dx} = -\sin x$$

$$= \int (1-u^2)\sin x \frac{1}{-\sin x}\,du = -\int 1 - u^2\,du$$

$$= -u + \frac{1}{3}u^3 + c = -\cos x + \frac{1}{3}\cos^3 x + c \qquad \text{check!}$$

moral: if there is an odd power want to do sub u = other trig function

Example

$$\int \sin^4 x \underbrace{\cos^3 x}_{\text{odd power}}\,dx \qquad \text{try: } u = \sin x \quad \frac{du}{dx} = \cos x$$

$$= \int \sin^4 x \cos^2 x \cdot \cos x\,dx = \int \sin^4 x(1-\sin^2 x)\cos x\,dx$$

$$= \int u^4(1-u^2)\,du = \int u^4 - u^6\,du = \frac{1}{5}u^5 - \frac{1}{7}u^7 + C$$

even powers $\int \sin^4 x \cos^2 x\,dx$? get everything in terms of $\sin(x)$ or $\cos(x)$, then use parts.

$$\int \sin^4 x(1-\sin^2 x)\,dx = \int \sin^4 x - \sin^6 x\,dx$$

$$\int \sin^6 x\,dx = \int \underbrace{\sin^5 x}_{u}\ \underbrace{\sin x}_{v'}\,dx$$

$u = \sin^5 x \quad u' = 5\sin^4 x \cdot \cos x$

$v' = \sin x \quad v = -\cos x$

$$= uv - \int u'v\,dx$$

$$-\sin^5 x\cos x + \int 5\sin^4 \cos^2 x\,dx = -\sin^5 x\cos x + 5\int \sin^4 x(1-\sin^2 x)\,dx$$

so $$\int \sin^6 x\,dx = -\sin^5 x\cos x + 5\int \sin^4 x\,dx - 5\int \sin^6 x\,dx$$

$$6\int \sin^6 x\,dx = -\sin^5 x\cos x + \underbrace{5\int \sin^4 x\,dx}_{\text{do by parts!}}$$

other trig functions

recall $\int \tan(x)\,dx = \int \frac{\sin x}{\cos x}\,dx$ $\quad u = \cos x \quad \frac{du}{dx} = -\sin x$ $\quad = \int \frac{\sin x}{u} \cdot \frac{-1}{\sin x}\,du$

$$= -\int \frac{1}{u}\,du = -\ln|u| + C = -\ln|\cos x| + C = \ln|\sec x| + C.$$

fact: $\int \sec(x)\,dx = \ln|\sec x + \tan x| + C.$

other trig function powers

$\int \tan^9 x \sec^6 x\,dx$

use:
- $\cos^2 x + \sin^2 x = 1 \longleftrightarrow 1 + \tan^2 x = \sec^2 x$
- $u = \sec x \quad \frac{du}{dx} = \sec x \tan x$
- $u = \tan x \quad \frac{du}{dx} = \sec^2 x.$
- parts