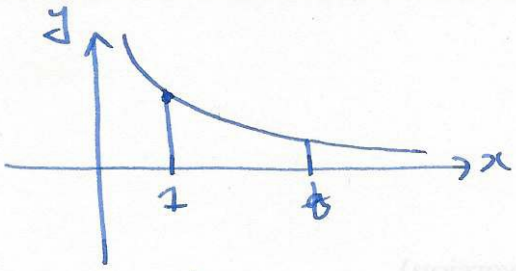
Example 

$y = \frac{1}{x}$ between 1 and ∞.
1 and t then let $t \to \infty$.

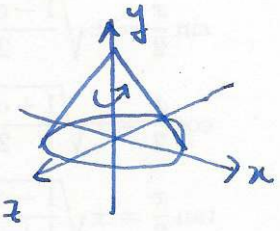
volume $V = \int_1^t \frac{\pi}{x^2}\,dx = \pi\left[-\frac{1}{x}\right]_1^t = \pi\left(1-\frac{1}{t}\right) \to \pi$ as $t \to \infty$.

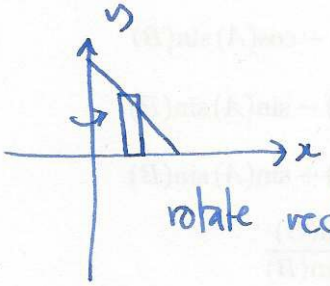
surface area $A = 2\pi\int_1^t \frac{1}{x}\underbrace{\sqrt{1+\frac{1}{x^4}}}_{>1}\,dx > 2\pi\int_1^t \frac{1}{x}\,dx = 2\pi\left[\ln|x|\right]_1^t$

$= 2\pi\ln(t) \to \infty$ as $t \to \infty$.

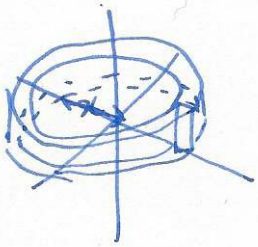
§6.4 Cylindrical shells

Volume of cone:





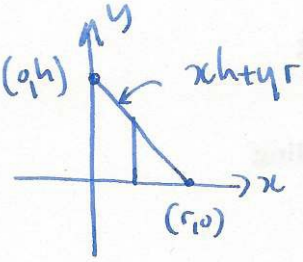
rotate rectangle around y-axis



volume of shell $V_i \approx 2\pi x y \Delta x$

total volume $V \approx \sum v_i = \sum 2\pi x_i y(x_i)\Delta x_i$

$$V = 2\pi\int_a^b x f(x)\,dx$$

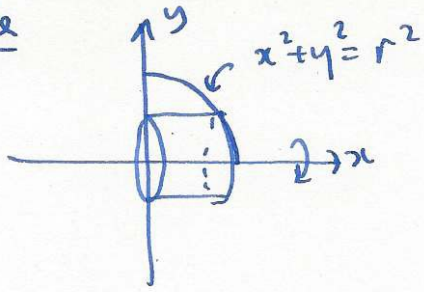
Example 

$$V = \int_0^r 2\pi x\left(\frac{hr-xh}{r}\right)dx$$

$$V = \frac{2\pi h}{r}\int_0^r x(r-x)\,dx = 2\pi\frac{h}{r}\int_0^r xr - x^2\,dx = 2\pi\frac{h}{r}\left[\frac{1}{2}x^2r - \frac{1}{3}x^3\right]_0^r$$

$$= 2\pi\frac{h}{r}\left(\frac{1}{2}r^3 - \frac{1}{3}r^3\right) = \frac{1}{3}\pi h r^2.$$

Example



volume of hemisphere: rotate horizontal rectangles around x-axis

$$V = \int 2\pi y f(y)\,dy$$

$$V = \int_0^r 2\pi y \sqrt{r^2-y^2}\,dy = 2\pi \left[\frac{2}{3}(r^2-y^2)^{3/2} \cdot -\frac{1}{2} \right]_0^r$$

$$= -\frac{2\pi}{3}(0-r^3) = \frac{2}{3}\pi r^3 \quad \left(\text{while sphere } \frac{4}{3}\pi r^3\right).$$

§7.21 Integration by parts.

"reverse product rule". product rule: $(uv)' = u'v + uv'$

$$\int (uv)'\,dx = \int u'v + uv'\,dx$$

$$uv = \int u'v\,dx + \int uv'\,dx$$

$$\boxed{\int uv'\,dx = uv - \int u'v\,dx}$$

Example

$$\int \underbrace{x}_{u} \underbrace{\sin(x)}_{v'}\,dx \qquad u = x \qquad u' = 1$$

$$v' = \sin x \qquad v = -\cos(x)$$

$$\int \underbrace{x}_{u} \underbrace{\sin(x)}_{v'}\,dx = \underbrace{x}_{u} \cdot \underbrace{(-\cos(x))}_{v} - \int \underbrace{(1)}_{u'} \cdot \underbrace{(-\cos(x))}_{v}\,dx$$

$$= -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x + c$$

check! $(-x\cos x + \sin x)' = -x \cdot -\sin x - \cos(x) + \cos x = x\sin x$ ✓.