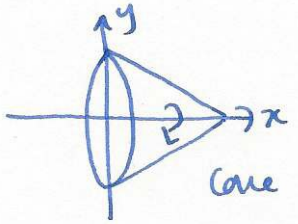
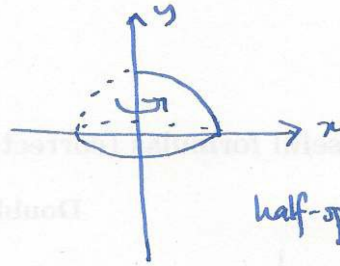
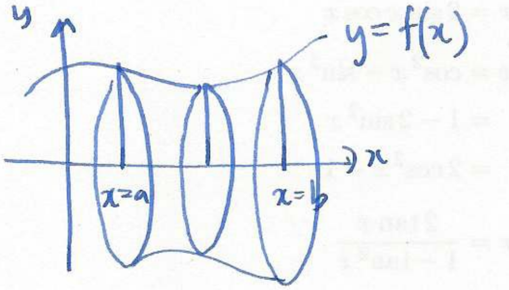
§6.3 Volume of revolution

Examples







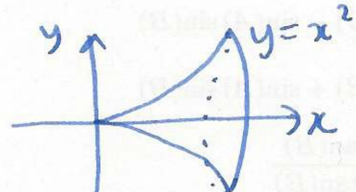
recall $V = \int_a^b A(x)\,dx$

$A(x)$ = area of vertical cross section

$= \pi y^2 = \pi (f(x))^2$

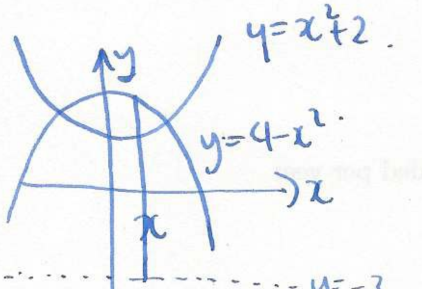
so volume $V = \int_a^b \pi (f(x))^2\,dx$

Example rotate $y=x^2$ about x-axis.



$$\pi \int_0^2 (x^2)^2\,dx = \pi \left[\frac{1}{5}x^5\right]_0^2 = \frac{32\pi}{5}$$

Example rotate region between $f(x) = x^2+2$ and $g(x) = 4-x^2$ about $y=-3$.



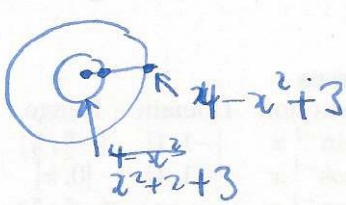
$V = \int_a^b A(x)\,dx$

a,b: find intersections:

$x^2+2 = 4-x^2$.

$2x^2 = 2 \quad x = \pm 1$.

$A(x)$ area of cross section.



$= \cancel{\pi((4-x^2)^2-(x^2+2)^2)}$

$$A(x) = \pi\left[(4-x^2+3)^2 - (x^2+2+3)^2\right].$$

$(7-x^2)^2 - (x^2+5)^2$.

$49 - 14x^2 + x^4 - x^4 - 10x^2 - 25$.

$24 - 24x^2$

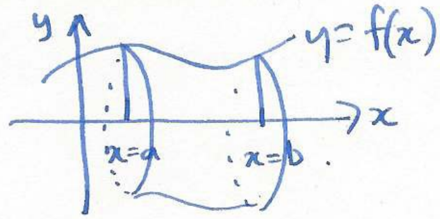
$24\pi \int_{-1}^{1} \cancel{24\pi}\ 1-x^2\,dx$

$24\pi\left[x - \frac{1}{3}x^3\right]_{-1}^{1} = \left(\frac{2}{3} - \left(-1+\frac{1}{3}\right)\right)24\pi$

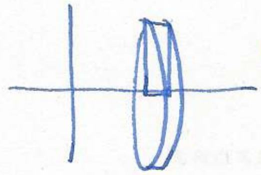
$\frac{4}{3}$.

$= 32\pi$.

Surface area:



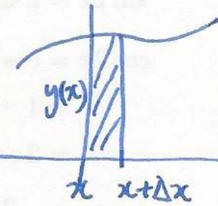
<u>wrong</u>:



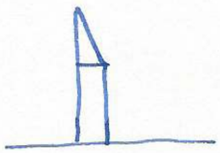
area of cylinder $\simeq 2\pi y \Delta x$ surface area $\simeq \sum 2\pi y(x_i)\Delta x_i$

$$2\pi\int_a^b y\,dx.$$

why is this wrong?

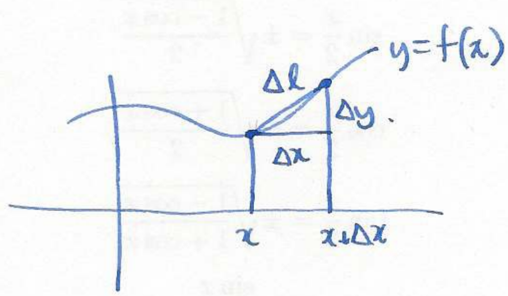


area of strip $\simeq y(x)\Delta x$. $\frac{\text{error}}{\text{area}} \to 0$ as $\Delta x \to 0$.



surface area $\neq 2\pi y(x)\Delta x$ $\frac{\text{error}}{\text{surface area}} \not\to 0$ as $\Delta x \to 0$.

right way



arc length: $\sum \Delta l$

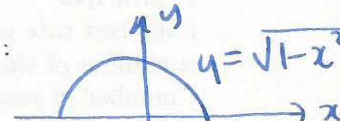
$$(\Delta l)^2 = (\Delta x)^2 + (\Delta y)^2$$

surface area $\simeq y\, 2\pi \Delta l$

total surface area: $\sum 2\pi y \Delta l = \sum 2\pi y(x_i)\sqrt{(\Delta x_i)^2 + (\Delta y(x_i))^2}$ ← doesn't look like integral!

$$= 2\pi \sum y(x_i)\sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2}\,\Delta x \longrightarrow \boxed{S.A. = 2\pi\int_a^b y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx}.$$

check: surface area of sphere



$y = \sqrt{1-x^2}$.

$$\frac{dy}{dx} = \frac{1}{2}(1-x^2)^{-1/2}(-2x).$$

$$2\pi\int_{-1}^{1}\sqrt{1-x^2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$2\pi\int_{-1}^{1}\sqrt{1-x^2}\sqrt{1+\frac{x^2}{1-x^2}}\,dx = 2\pi\int_{-1}^{1}\sqrt{1-x^2}\sqrt{\frac{1-x^2+x^2}{1-x^2}}\,dx = 2\pi\int_{-1}^{1}\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}\,dx$$

$$= 2\pi\int_{-1}^{1}dx = 2\pi\,[x]_{-1}^{1} = 4\pi.$$