MTH 232 Calculus 2

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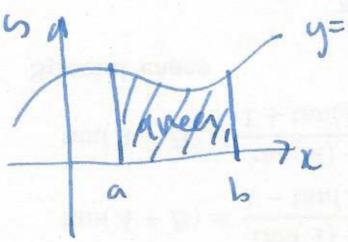
- students with disabilities

<u>Text</u>: Calculus, early transcendentals, Rogawski

<u>HW</u>: webworks / Matlab projects.

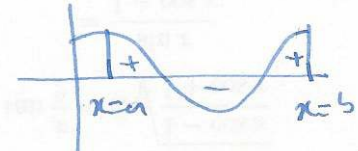
§5.2 Definite integral

<u>intuition</u>:



$$\int_a^b f(x)\,dx = \text{area under the curve } y=f(x) \text{ between } x=a \text{ and } x=b$$

<u>note</u>: signed area:

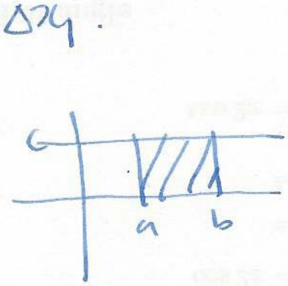


formal defn: Riemann sum $R(f,P,C)$ P partition of $[a,b]$ $x_0\ x_1 \dots x_n$

$R(f,P,C) = \sum f(c_i)\Delta x_i \quad \Delta x_i = |x_{i+1} - x_i|$ C choice of $c_i \in [x_i, x_{i+1}]$

$$\int_a^b f(x)\,dx = \lim_{\|P\|\to 0} R(f,P,C) \qquad \|P\| = \max \Delta x_i .$$

<u>Useful properties</u> $\int_a^b c\,dx = c(b-a)$



$$\int_a^b f(x) + g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$$

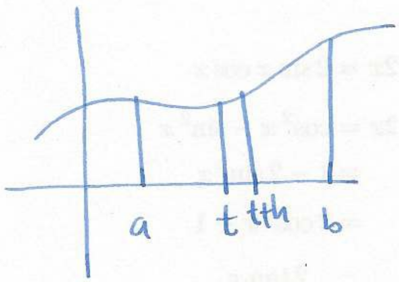
reversing limits: $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

0-length interval $\int_a^a f(x)\,dx = 0$ adjacent. $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

§5.3 Fundamental theorem of calculus. I

Thm (FTC ①) suppose $f(x)$ cts on $[a,b]$ and $F(x)$ is an anti-derivative for $f(x)$. [i.e. $F'(x) = f(x)$]. Then $\int_a^b f(x)\,dx = F(b) - F(a)$.

intuition



consider $\int_a^t f(x)\,dx$

Q: what is rate of change wrt t?

$$\frac{d}{dt}\left(\int_a^t f(x)\,dx\right) = \lim_{h\to 0} \frac{\int_a^{t+h} f(x)\,dx - \int_a^t f(x)\,dx}{h} = \lim_{h\to 0} \frac{\int_t^{t+h} f(x)\,dx}{h}$$

$$\left[\begin{array}{l}\text{recall} \\ f'(t) = \lim_{h\to 0} \frac{f(t+h)-f(t)}{h}\end{array}\right] \underset{\text{approx}}{\approx} \frac{\text{area of rectangle } \cancel{f(t)} \times h}{h} = f(t)$$

i.e. $\int_a^t f(x)\,dx$ is an anti-derivative for $f(x)$, so $\int_a^t f(x) = F(x) + C$

what is the constant? $t=a$: $\int_a^a f(x)\,dx = 0 = F(a) + C$

$$\Rightarrow \quad C = -F(a)$$

so $\int_a^t f(x)\,dx = F(t) - F(a)$ $\square$.

Example. $$\int_2^3 \sqrt{x} + \frac{1}{x} + \sin(x)\,dx = \left[\frac{2x^{3/2}}{3} + \ln|x| - \cos(x)\right]_2^3.$$

$$= \frac{2}{3}(3)^{4/3} + \ln(3) - \cos(3) - \left(\frac{2}{3}(2)^{4/3} + \ln(2) - \cos(2)\right).$$

§5.4 Fundamental theorem of calculus II

Thm (FTC ②) Let $f(x)$ be cts on $[a,b]$, then $A(x) = \int_a^x f(t)\,dt$ is an anti-derivative for $f(x)$, i.e. $A'(x) = f(x) = \frac{dA}{dx} \leftrightarrow \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

Furthermore: $A(a) = 0$.

Example $\int_0^x e^{-t^2}\,dt$ ← a function with derivative e^{-x^2} (*)

Example what about $\int_0^{x^2} \sin(t)\,dt$ ← function of a function

to find $\frac{d}{dx}\int_0^{x^2}\sin(t)\,dt$ set $A(x) = \int_0^x \sin(t)\,dt$ then $A'(x) = \sin(x)$

so $$\frac{d}{dx}\int_0^{x^2}\sin(t)\,dt = \frac{d}{dx}\left(A(x^2)\right) \underset{\text{chain rule}}{=} A'(x^2)\cdot(x^2)' = A(x^2)\cdot 2x.$$
$$= \sin(x^2)\cdot 2x.$$

(*) Aside: when people say "not every formula can be integrated" what do they mean?

$f(x)$ cts then $A(x) = \int_0^x f(t)\,dt$ is an integral for $f(x)$, but might not be able to write it down as a formula involving basic functions.

Analogy
- $\sqrt{2}$ is a number, but not a fraction $\sqrt{2} = 1.414\cdots$
- $x^5 - x - 1$ has 1 real root which can not be written as an expression involving rational numbers and fractional powers [Galois theory]
- e^{-x^2} has an integral that can't be written as a combination of elementary functions [differential Galois theory].

Useful rules

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\tan^{-1}(x)$	$\frac{1}{1+x^2}$
e^x	e^x
$\ln(x)$	$1/x$

$f(x)$	$\int f(x)\,dx$	
x^n	$\frac{x^{n+1}}{n+1}$	$(n \neq -1)$
$\sin(x)$	$-\cos(x)$	
$\cos(x)$	$\sin(x)$	
$\frac{1}{x} = x^{-1}$	$\ln(x)$	
e^x	e^x	

general rules

f	f'
$kf(x)$	$kf'(x)$
$f(x)+g(x)$	$f'(x)+g'(x)$
$f(x)g(x)$	$f'(x)g(x)+f(x)g'(x)$
$f(x)/g(x)$	$\frac{gf'-fg'}{x}$

$f(g(x)) = f'(g(x))g'(x)$.

f	$\int f\,dx$
$kf(x)$	$k\int f(x)\,dx$
$f(x)+g(x)$	$\int f(x)+g(x)\,dx$