Math 232 Calculus 2 Spring 15 Midterm 1b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Find $\displaystyle\int \frac{e^{3x}}{e^{3x}-1}\,dx$.

$u = e^{3x} - 1$

$\frac{du}{dx} = 3e^{3x}$

$$\int \frac{e^{3x}}{u}\,\frac{dx}{du}\,du$$

$$\int \frac{e^{3x}}{u}\cdot\frac{1}{3e^{3x}}\,du$$

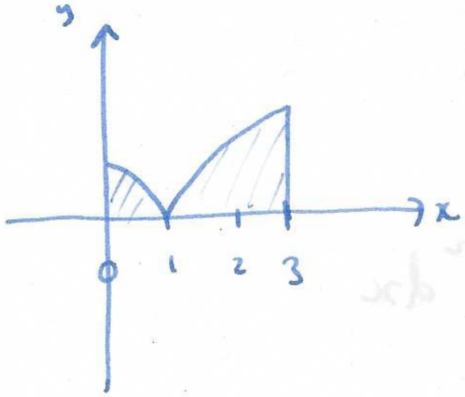
$$\frac{1}{3}\int \frac{1}{u}\,du$$

$$\frac{1}{3}\ln|u| + c$$

$$\frac{1}{3}\ln\left|e^{3x}-1\right| + c$$

$1-x^2=0 \Rightarrow x=\pm 1$

(2) (10 points) Find $\int_0^3 \left|1-x^2\right| \, dx$. Draw a picture of the region.



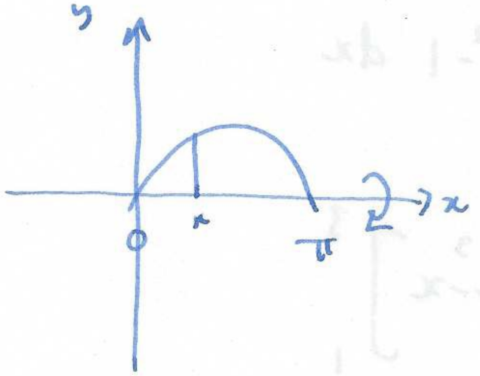
$$\int_0^1 1-x^2 \, dx + \int_1^3 x^2-1 \, dx$$

$$\left[x-\frac{1}{3}x^3\right]_0^1 + \left[\frac{1}{3}x^3-x\right]_1^3$$

$$1-\frac{1}{3} + 9-3-\frac{1}{3}+1$$

$$7\frac{1}{3}$$

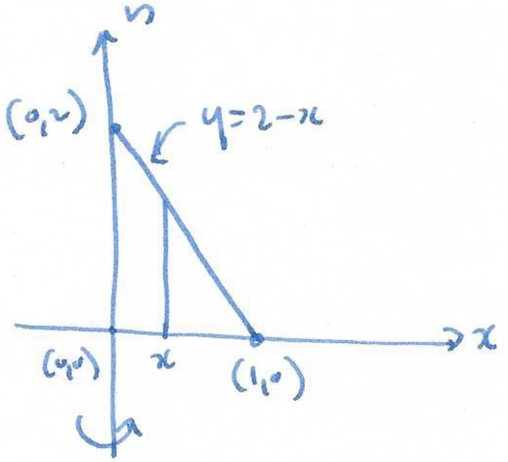
(3) (10 points) Draw a picture of the region bounded by the curve $y = 2\sin(x)$, for $0 \le x \le \pi$ and $y \ge 0$. Write down an integral to give you the volume of revolution of this region about the x-axis. DO NOT EVALUATE THIS INTEGRAL.



discs : $A(x) = \pi y^2$

$$\int_0^{\pi} \pi \left(2\sin(x)\right)^2 dx$$

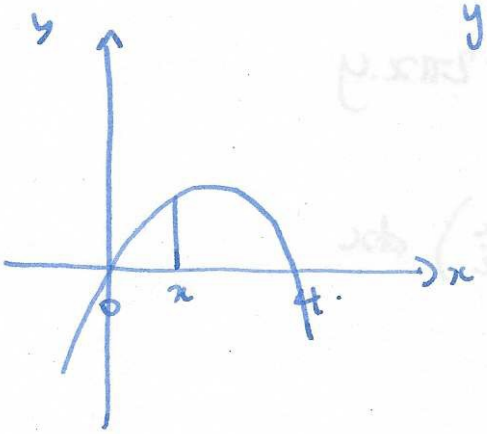
(4) (10 points) Use shells to write down an integral for the volume of the cone formed by rotating the triangle with vertices $(0, 0)$, $(0, 2)$ and $(1, 0)$ about the y-axis. DO NOT EVALUATE THIS INTEGRAL.



shells: $A(x) = 2\pi x y$

$$\int_0^1 2\pi x\,(2-x)\,dx$$

(5) (10 points) Consider the subset of the plane bounded by $y = 4x - x^2$ in the first quadrant (i.e. $x \geq 0$ and $y \geq 0$). Find the volume of revolution of the 3-dimensional shape formed by rotating this region around the x-axis.



$$y = 4x - x^2 = x(4-x)$$

discs : $A(x) = \pi y^2$

$$\int_0^4 \pi (4x - x^2)^2 \, dx$$

$$\pi \int_0^4 16x^2 - 8x^3 + x^4 \, dx$$

$$\pi \left[\frac{16}{3} x^3 - 2x^4 + \frac{1}{5} x^5 \right]_0^4$$

$$\frac{1024}{3} - 512 + \frac{1024}{5}$$

$$\pi 34\tfrac{1}{6} \approx 107.33$$

(6) (10 points) Find $\int \underbrace{\sqrt{x}}_{v}\underbrace{\ln(3x)}_{u}\, dx$.

$$\int uv'\,dx = uv - \int u'v\,dx$$

$u = \ln(3x)$ $\quad u' = \frac{1}{x}$

$v = x^{1/2}$ $\quad v = \frac{2x^{3/2}}{3}$

$$= \frac{2}{3}x^{3/2}\ln(3x) - \int \frac{1}{x}\frac{2}{3}x^{3/2}\,dx$$

$$\frac{2}{3}x^{3/2}\ln(3x) - \int \frac{2}{3}x^{1/2}\,dx$$

$$\frac{2}{3}x^{3/2}\ln(3x) - \frac{4}{9}x^{3/2} + c$$

(7) (10 points) Find $\int \underbrace{e^{-2x}}_{u} \underbrace{\sin(x)}_{v}\, dx$.

$\int uv'\,dx = uv - \int u'v\,dx$

$u = e^{-2x}$ $u' = -2e^{-2x}$

$v' = \sin(x)$ $v = -\cos(x)$

$\int e^{-2x}\sin(x)\,dx = -e^{-2x}\cos(x) - \int \underbrace{2e^{-2x}}_{u} \underbrace{\cos(x)}_{v'} dx$

$u = 2e^{-2x}$ $u' = -4e^{-2x}$

$v' = \cos x$ $v = \sin x$

$= -e^{-2x}\cos x - 2e^{-2x}\sin x + \int -4e^{-2x}\sin x\,dx$

$5\int e^{-2x}\sin x\,dx = -e^{-2x}(\cos x + 2\sin x) + c$

$\int e^{-2x}\sin x\,dx = -e^{-2x}\left(\frac{1}{5}\cos x + \frac{2}{5}\sin x\right) + c$

(8) Find $\int_0^{\pi/2} \sin^5 x \, dx$. $= \int_0^{\pi/2} \sin^4 x \sin x \, dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$$= \int_0^{\pi/2} (1-\cos^2 x)^2 \sin x \, dx$$

$$= -\int_1^0 (1-u^2)^2 \sin x \frac{dx}{du} \, du$$

$$= -\int_1^0 (1-u^2)^2 \sin x \frac{1}{\sin x} \, du$$

$$= -\int_1^0 1 - 2u^2 + u^4 \, du$$

$$= -\left[u - \frac{2}{3}u^3 + \frac{1}{5}u^5 \right]_1^0$$

$$= 1 - \frac{2}{3} + \frac{1}{5} = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$$

(9) Find $\int \sin(3x)\sin(4x)\,dx$.

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\cos(A+B) - \cos(A-B) = -2\sin A\sin B$

$= \int \frac{1}{2}\cos(3x-4x) - \frac{1}{2}\cos(3x+4x)\;dx$

$= \frac{1}{2}\int \cos(x) - \cos(7x)\;dx$

$\frac{1}{2}\sin x - \frac{1}{7}\sin 7x + c$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

(10) Find $\int \sqrt{9 - x^2}\, dx$.

$x = 3\sin u \leftrightarrow \quad \sin u = \frac{x}{3} \quad \cos u = \sqrt{1 - x^2/9}$

$\frac{dx}{du} = 3\cos u$

$$\int \sqrt{9 - 9\sin^2 u}\, \frac{dx}{du}\, du$$

$$3\int \sqrt{1 - \sin^2 u}\; 3\cos u\, du$$

$$9\int \cos^2 u\, du$$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$$9\int \frac{1}{2}\cos 2u - \frac{1}{2}\, du$$

$$\frac{9}{2} \cdot \frac{1}{2}\underbrace{\sin 2u}_{\substack{\shortparallel \\ 2\sin u \cos u}} - \frac{9}{2}u + C$$

$$\frac{9}{2}\, x\sqrt{1 - x^2/9} - \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + C$$