MAT 233 FINAL EXAM SUMMER 12

1.a) Find the volume of the parallelopiped spanned by the vectors $\mathbf{u}=2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ and $\mathbf{v}=-4\mathbf{i}+\mathbf{j}$ and $\mathbf{w}=-3\mathbf{i}+\mathbf{k}$

b) What is the angle between $\mathbf{u}\times\mathbf{v}$ and $\mathbf{w}$?

2. Let $r(t) = (3\sin t, 3\cos t, 10 - \frac{1}{2}t^2)$ give the position of a particle as a function of time and answer each of the following:

a)The velocity $\mathbf{v}$(t) ?

b)The acceleration $\mathbf{a}$(t) ?

c) The projection of the acceleration in the direction of the velocity $\mathbf{a}(t)_T$

3. Let $\phi(x, y, z) = z^2 \ln xy$

a) Find the vector field $\mathbf{F}=\nabla\phi$.

b) Compute the curl of $\mathbf{F}$:$\nabla X\mathbf{F}$

c) Compute the line integral of $\mathbf{F}$ over any path starting at the point (1,1,2) and ending at the point (e,e,4).

4. Let $\mathbf{F}$(x,y,z)=$(3x, xz-5, zx)$ and let S be the surface of rectangular box $0 \leq x \leq 4$, $0 \leq y \leq 4$ and $0 \leq z \leq 3$. Compute:

$$\int_S \mathbf{F} \cdot \mathbf{dS}$$

Hint:Use the divergence theorem

5. Use Green's Theorem to compute $\int_C 5\cos x\,dx + xy\,dy$ where C is the the curve that bounds the region between $y = \frac{1}{2}x$ and $y = \sqrt{x}$. Sketch the region.

6. Let $\Phi(u, v) = (u\cos v, u\sin v, v)$. Answer each of the following:

a) Find $T_u = \frac{\partial \Phi}{\partial u}$ and $T_v = \frac{\partial \Phi}{\partial v}$

b) Find n at the point $(3, 0, 2\pi)$. Hint $\Phi(3, 2\pi) = (3, 0, 2\pi)$

c) Give the equation of the tangent plane at $(3, 0, 2\pi)$

7. Find the criticals point of $f(x, y) = x^2 - 5x - 3y^2 + 4y - 22$ and apply the second derivative test to determine what kind.

7...continued

8. Integrate $f(x, y, z) = x + 2z$ over the region bounded by $0 \leq x^2 + y^2 \leq 3$ and$-2 \leq z \leq 4$.
(Hint use cylindrical coordinates).