MATH 233 - FINAL EXAM

College of Staten Island   Fall 2012   Pribitkin

Print Name: ____________________________________

Instructions: You are permitted two $8\frac{1}{2}''$ X $11''$ sheets of paper with your own handwritten/typed notes on both sides. Calculators are allowed, but all answers must be explained fully (just as we did in class). **Please write clearly.** There are five pages. The time limit is 115 minutes. Good luck!

1. (6 points each) Consider the three points O, $P = (1, 1, 0)$, and $Q = (0, 1, 1)$.

   (a) Find the equation of the plane passing through O, P, and Q.

   (b) Now find the area of the triangle with vertices at O, P, and Q.

2. (6 points each) Consider the curve parametrized by $\mathbf{r}(t) = \langle \cos 8t, \sin 8t, 6t \rangle$.

   (a) Compute its arc length over the interval $0 \le t \le \pi$.

   (b) Now find an arc length parametrization for the curve.

3. (5 points each) For each of the following, find the limit or determine that it does not exist.

(a) $\lim_{(x,y)\to(2,1)} e^{x-y}\sin(3\pi xy)$

(b) $\lim_{(x,y)\to(0,0)} \dfrac{x^3y}{x^4+y^4}$

4. (6 points each) Let $f(x,y)=\ln(1+x^2y^4)$.

(a) Calculate the gradient of $f(x,y)$.

(b) Now find the directional derivative of $f(x,y)$ in the direction of $\mathbf{v}=\langle 12,5\rangle$ at the point $(1,-1)$.

5. (6 points each) Consider the function $g(x, y) = x^2 + 2xy + 3y^2$.

(a) Find the critical points of $g(x, y)$.

(b) Find the local extrema and saddle points of $g(x, y)$.

(c) Now find the global extreme values of $g(x, y)$ on the unit square $0 \leq x \leq 1, 0 \leq y \leq 1$.

6. (12 points) Evaluate $\int_0^1 \int_{\sqrt{y}}^1 \frac{3}{(x^3+1)^5}\, dx\, dy$.

7. (12 points) Find the volume of the solid that lies under the surface $z = e^{-x^2-y^2}$ and above the region in the xy-plane bounded by the circle $x^2 + y^2 = a^2$.

8. (12 points) Let W be the solid region that lies inside the unit sphere $x^2 + y^2 + z^2 = 1$ and above the plane $z = 0$. The density of the solid is $\delta(x, y, z) = 1 + \sqrt{x^2 + y^2 + z^2}$. Find the mass of W.

EXTRA CREDIT: Now find the center of mass of W.