Mth 233/9221

Your Name:

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CIIIfex**A**

FINAL EXAM

YOU MUST SHOW WORK AND GIVE EXPLANATIONS OF YOUR ANSWERS; ANSWERS ALONE EARN NO CREDIT! IF YOU ARE USING FORMULAS WRITE WHAT FORMULAS YOU ARE USING. DO ALL YOUR WORK AND BOX THE ANSWERS ON THIS TEST PAPER AND IN THE APPROPRIATE PLACES. IF YOU NEED EXTRA SPACE, WRITE ON BACK OF THE PAGE(S) AND INDICATE THAT YOU ARE DOING SO. IF FOR ANY PARTS YOU USE A CALCULATOR, STATE SO. YOU DO HAVE TO SHOW STEPS IN SOLVING A PROBLEM, EVEN IF YOU USE A CALCULATOR.

GOOD LUCK!

1) Given three points in 3D: $A(2, 0, a), B(0, 1, a), C(-1 - 1, 0)$, where a is an arbitrary given constant.

a) Calculate the vector product $\vec{AB} \times \vec{AC}$.

b) Calculate the scalar product $\vec{AB} \cdot \vec{AC}$.

c) For what values of a are vectors $\vec{AB}$ and $\vec{AC}$

i) perpendicular, ii) parallel ?

d) Write equation of the plane that contains points A,B,C.

2) Find a function $f(x, y)$ that satisfies the equation $\nabla f = (y^2, x)$ or prove that such a function does not exist. (You may assume that the function has partial derivatives of any order.)

3) Express partial derivatives $\partial f/\partial r, \partial f/\partial \theta$ of a function $f(x, y, z)$ in terms of $\partial f/\partial x, \partial f/\partial y, \partial f/\partial z$ where (r, θ, z) are cylindrical coordinates.

4) Given points A(0,1), B(1,2) and C(3,0) (in 2D) and the curve γ that consists of the portion of the parabola $y = 1 + x^2$ between points A and B and the straight segment connecting B and C, find the line integral

$$\int_{\gamma} \vec{F}\, ds$$

where $\vec{F}(x,y) = (x^2 - y, y^2 + x)$ and the curve is traversed in the direction from A to B to C. What would be the value of the integral if the orientation of curve γ were from C to B to A?

5) The function $f(x, y)$ is defined as follows on the domain $[0,1] \times [0,1]$:

$$f(x,y) = \begin{cases} \frac{x}{\arctan(1/y^2)}, & \text{if} \quad y \neq 0 \\ 2x\pi, & \text{if} \quad y = 0 \end{cases}$$

Find all the points where the function is continuous (or not continuous) in its domain.

6) Find the volume of the region bounded by the hyperbolic cylinders $xy = 1, xy = 9, xz = 4, xz = 36, yz = 25, yz = 49$. [Hint: Make an appropriate change of variables and use the change of variables theorem to compute it.]

7) Find the volume of the region bounded by the sphere $x^2 + y^2 + z^2 = a^2$ (a is a given constant) and the plane $z = b$, where constant b is such that $a > b > 0$, using either cylindrical or spherical coordinates.