§15.4 Integration in polar, cylindrical, and spherical coordinates

<u>recall</u>: double integral over regions $\iint_D f(x,y)\,dA$

some regions easier to describe in polars.

$0 \le \theta \le \pi/2$
$0 \le r \le 1$

$0 \le \theta \le \pi/2$
$1 \le r \le 2$

<u>useful fact</u>: in polar coordinates $\iint_D f(x,y)\,dA = \iint_D f(r,\theta)\, r\, dr\, d\theta$

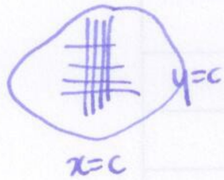
$\uparrow$ $f(x(r,\theta), y(r,\theta))$

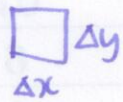
i.e. $dA = dx\,dy$ in cartesians

$\boxed{dA = r\,dr\,d\theta}$ in polars

<u>warning</u>: do not forget this r

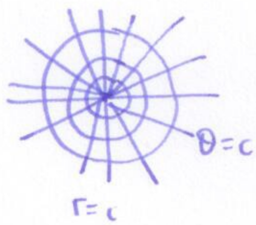
<u>idea</u>: <u>cartesians</u>:





area $\Delta A = \Delta x \Delta y$

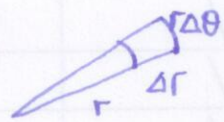
<u>polars</u>:



"squares" are different sizes!

area $\Delta A \approx r\Delta r \Delta\theta$

area of wedge



~~area is $\approx r\Delta r\Delta\theta$.~~

~~$\approx (r+\Delta r)\Delta\theta - r\Delta\theta \approx \Delta r \Delta\theta$~~

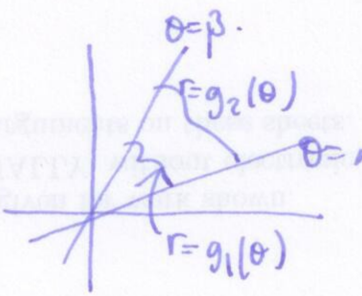
area $\pi r^2 \frac{\theta}{2\pi} = \frac{1}{2} r^2\theta$

$$\approx \frac{1}{2}(r+\Delta r)^2 \Delta\theta - \frac{1}{2} r^2 \Delta\theta$$

$$= \frac{1}{2}\Delta\theta\left(\cancel{r^2} + 2\Delta r + \Delta r^2 - \cancel{r^2}\right) \approx r\Delta r \Delta\theta.$$

Describing regions

$$\int_\alpha^\beta \int_{g_1(\theta)}^{g_2(\theta)} f(r,\theta)\, r\, dr\, d\theta$$



$$\int_a^b \int_{h_1(r)}^{h_2(r)} f(r,\theta)\, r\, d\theta\, dr$$