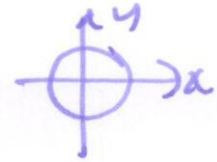
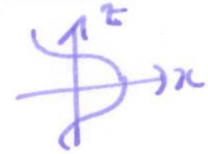
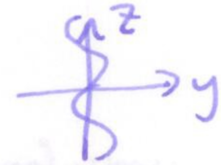
**projections**: set one coordinate to zero



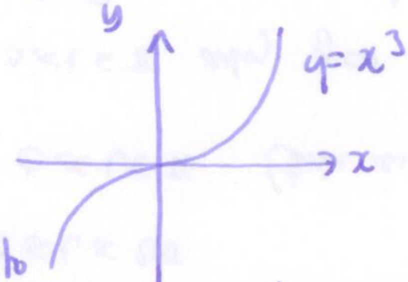

$\langle \cos t, \sin t, 0 \rangle$ projection onto $xy$-plane

$\langle \cos t, 0, t \rangle$ $xz$-plane

$\langle 0, \sin t, t \rangle$ $yz$-plane

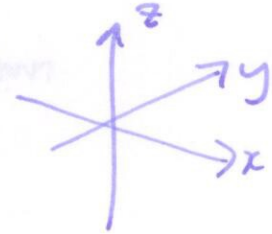
## Finding parameterizations

**examples** ① $y = x^3$




$\underline{r}(t) - \langle t, t^3 \rangle$

② circle of radius 4 ~~in~~ parallel to $xz$-plane with center $(1,2,3)$




unit circle in $xy$-plane: $\underline{r}(t) = \langle \cos t, \sin t, 0 \rangle$

$xz$-plane: $\underline{r}(t) = \langle \cos t, 0, \sin t \rangle$

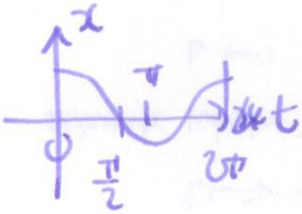
radius 4: $\underline{r}(t) = \langle 4\cos t, 0, 4\sin t \rangle$

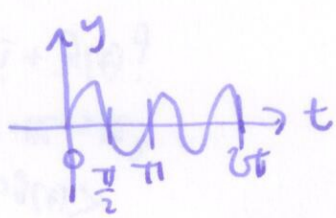
translate: $\underline{r}(t) = \langle 4\cos t + 1, 2, 4\sin t + 3 \rangle$.

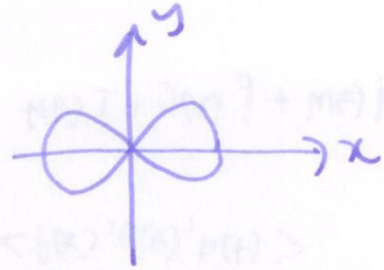
## Drawing parameterized curves

**Example** $\underline{r}(t) = \langle \cos t, \sin 2t \rangle$ ⊛

try drawing $(x,t)$, $(y,t)$ pictures.




Q: can you find an equation in $x, y$ satisfied by ⊛?

## §13.2 Calculus with vector valued functions

**Limits**: Def$^{\underline{n}}$ A vector valued function $\underline{r}(t)$ has a limit $\underline{v}$ at $t_0$ if $\lim_{t \to t_0} \|\underline{r}(t) - \underline{v}\| = 0$. If this happens we say $\lim_{t \to t_0} \underline{r}(t) = \underline{v}$

<u>Thm</u> Limits are computed pointwise:

$\underline{r}(t) = \langle x(t), y(t), z(t) \rangle$ has a limit as $t \to t_0$ iff each component has a limit, and $\lim_{t\to t_0} \underline{r}(t) = \langle \lim_{t\to t_0} x(t), \lim_{t\to t_0} y(t), \lim_{t\to t_0} z(t) \rangle$

<u>Proof</u> (sketch) $\|\underline{r}(t) - \underline{v}\|^2 = (x(t)-v_1)^2 + (y(t)-v_2)^2 + (z(t)-v_3)^2$

LHS $\to 0$ iff each term in RHS $\to 0$ $\square$.

<u>Example</u> find $\lim_{t\to 0} \underline{r}(t)$ where $\underline{r}(t) = \langle t^2, t, \frac{\sin(t)}{t} \rangle$

$\lim_{t\to 0} \underline{r}(t) = \langle \lim_{t\to 0} t^2, \lim_{t\to 0} t, \lim_{t\to 0} \frac{\sin t}{t} \rangle = \langle 0, 0, 1 \rangle$

<u>Continuity</u>:

<u>Defn</u> $\underline{r}(t)$ is continuous iff all of its components are continuous.

<u>Differentiation</u>:

<u>Defn</u> The derivative of $\underline{r}(t)$ is

$$\underline{r}'(t) = \frac{d\underline{r}}{dt} = \lim_{h\to 0} \frac{\underline{r}(t+h) - \underline{r}(t)}{h}$$

if this limit exists we say that $\underline{r}(t)$ is <u>differentiable</u>.

<u>Thm</u> Derivatives are computed pointwise:

$\underline{r}(t) = \langle x(t), y(t), z(t) \rangle$, then $\underline{r}'(t) = \frac{d\underline{r}}{dt} = \langle x'(t), y'(t), z'(t) \rangle$

<u>Proof</u> derivative defined as a limit, limits are computed pointwise $\square$.

<u>Example</u> $\underline{r}(t) = \langle \cos t, \sin t, t \rangle$

$\underline{r}'(t) = \langle -\sin t, \cos t, 1 \rangle$

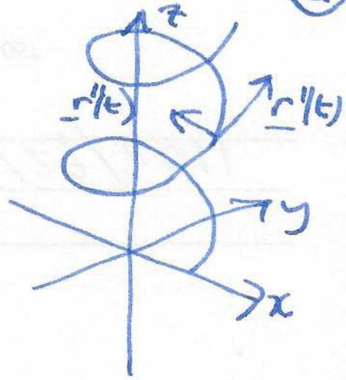
Higher derivatives: $\underline{r}''(t) = \frac{d^2\underline{r}}{dt^2} = \lim_{h\to 0} \frac{\underline{r}'(t+h) - \underline{r}'(t)}{h}$

<u>Example</u> $\underline{r}''(t) = \langle -\cos t, -\sin t, 0 \rangle$

if $\underline{r}(t) =$ position

then $\underline{r}'(t) =$ velocity

$\underline{r}''(t) =$ acceleration.




<u>Differentiation rules</u> ( $\underline{r}(t), \underline{r}_1(t), \underline{r}_2(t)$ differentiable)

- sums : $\left(\underline{r}_1(t) + \underline{r}_2(t)\right)' = \underline{r}_1'(t) + \underline{r}_2'(t)$
- scalar mult : $\left(c\,\underline{r}(t)\right)' = c\,\underline{r}'(t)$ $\quad c$ constant
- product rule : if $f(t)$ differentiable scalar function, then

$$\left(f(t)\underline{r}(t)\right)' = f(t)\underline{r}'(t) + f'(t)\underline{r}(t)$$

- chain rule : $\frac{d}{dt}\left(\underline{r}(f(t))\right) = f'(t)\,\underline{r}'(f(t))$.

<u>Warning</u> : $\left.\begin{matrix} f(\underline{r}(t)) \\ \underline{r}_1(\underline{r}_2(t)) \end{matrix}\right\}$ doesn't make sense!

<u>Proof</u> (sketch) everything is computed componentwise

product rule : $f(t)\underline{r}(t) = f(t) < r_1(t), r_2(t), r_3(t) >$

$$= < f(t) r_1(t), f(t) r_2(t), f(t) r_3(t) >$$

$$\left(f(t)\underline{r}(t)\right)' = < (f(t)r_1(t))', (f(t)r_2(t))', (f(t)r_3(t))' >$$

$$= < f'r_1 + fr_1', f'r_2 + fr_2', f'r_3 + fr_3' >$$

$$= < f'r_1, f'r_2, f'r_3 > + < fr_1', fr_2', fr_3' >$$

$$= f' < r_1, r_2, r_3 > + f < r_1', r_2', r_3' >$$

$$= f'(t)\underline{r}(t) + f(t)\underline{r}'(t).$$

others similarly □.