Math 233 Calculus 3 Fall 15 Sample Midterm 2

(1) You are standing on a surface given by the equation $z = 4x^2 - 2xy - y^2$. If you're standing at the point $(1, 2, -4)$, in which direction is the fastest way up?

(2) The temperature in the solar system is given by

$$T(x, y, z) = \frac{10^5}{x^2 + y^2 + z^2}$$

If a comet travels along the path $\mathbf{r}(t) = (2t, t^2 - 16, t)$, use the chain rule to determine how fast the temperature is changing when $t = 2$.

(3) Find the critical points of the following functions, and use the second derivative test to classify them, if possible.

(a)
$$f(x, y) = x^3 - 6xy + y^3$$

(b)
$$f(x, y) = 3xe^y - e^x$$

(c)
$$f(x, y) = 2x \ln(x + y)$$

(4) Find the extreme values of $f(x, y) = 2x^2 - 4y^2$ on the square $0 \leq x \leq 1, 0 \leq y \leq 1$.

(5) Use Lagrange multipliers to find the minimum and maximum values of $x^2y + 2x + y$ subject to $xy = 4$.

(6) Use Lagrange multipliers to find the dimensions of the cylindrical tin can of volume V with least surface area.

(7) Integrate the function $f(x, y) = xy$ over the triangle in the xy-plane with vertices $(0, 4), (1, 0)$ and $(2, 4)$.

(8) Evaluate the following integral by changing the order of integration.:

$$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \frac{y}{(1 + x^2 + y^2)^2} dxdy$$

(9) Write down limits for an integral over the tetrahedron with vertices $(0,0,0)$, $(0,1,1)$, $(1,1,0)$ and $(1,1,1)$.

(10) Write down limits for the following integrals.
  (a) The integral over the region in the octant $x \geq 0, y \leq 0, z \leq 0$ inside the cylinder $x^2 + y^2 = 4$ and the ellipsoid $2x^2 + 2y^2 + z^2 = 4$.
  (b) The integral over region with $y \leq 0$, which lies below the negative cone $z^2 = 3x^2 + 3y^2$ with $z \leq 0$, and inside the sphere of radius 5.
  (c) The integral over the tetrahedron with vertices $(0,0,0)$, $(0,1,0)$, $(0,1,1)$ and $(1,1,1)$.

(11) Find the volume of the solid contained in the cylinder $x^2 + y^2 = 9$, below the surface $z = 2(x+y)^2$ and above the surface $z = -(x-y)^2$.

(12) Use spherical coordinates to evaluate the following integral.

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{\sqrt{9-x^2-y^2}} e^{-(x^2+y^2+z^2)^{3/2}} dz dy dx$$

(13) Let $f(x,y,z) = e^y + xz$. Evaluate

$$\int_C f ds,$$

where C is the straight line path from $(-1,2,-2)$ to $(3,5,4)$.

(14) Show that the vector field $\mathbf{F} = \langle y^2, x, -z \rangle$ is not conservative. Evaluate

$$\int_C \mathbf{F}.\mathbf{ds}$$

where C is the circle of radius 3 in the plane $z = 1$ centered on the z-axis.

(15) Show that the vector field $\mathbf{F} = \langle ze^{xz}, -z\sin(yz), xe^{xz} - y\sin(yz) \rangle$ is conservative, and find a function $f(x,y,z)$ such that $\nabla f = \mathbf{F}$. Evaluate

$$\int_C \mathbf{F}.\mathbf{ds}$$

where C is the curve formed by the intersection of the plane $z = 3x + 2y$ with the sphere of radius 25 in the positive octant, oriented anticlockwise around the z-axis.