Math 233 Calculus 3 Fall 15 Midterm 1b

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator without symbolic algebra capabilities, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

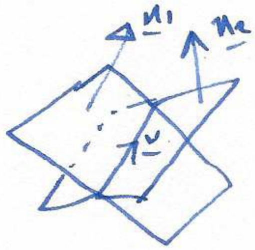
(1) (10 points) Find the angle between the two vectors $\langle 1, 2, -5\rangle$ and $\langle -2, 2, 3\rangle$.

$$\underline{u} \cdot \underline{v} = \|\underline{u}\| \, \|\underline{v}\| \cos\theta$$

$$-2 + 4 - 15 = \sqrt{1+4+25}\,\sqrt{4+4+9}\,\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{-13}{\sqrt{30}\,\sqrt{17}}\right) \approx 2.184$$

(2) (10 points) Find a parametric equation for the line of intersection of the two planes $2x - 2y + z = 2$ and $x + y - 3z = 1$.



$$\underline{v} = \underline{n}_1 \times \underline{n}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & -2 & 1 \\ 1 & 1 & -3 \end{vmatrix} = \langle 5, 7, 4 \rangle$$

find point of intersection, try $z=0$:

$$\left.\begin{aligned} 2x - 2y &= 2 \quad ① \\ x + y &= 1 \quad ② \end{aligned}\right\} \quad ① - 2② : \quad -4y = 0 \qquad \begin{aligned} y &= 0 \\ x &= 1 \end{aligned}$$

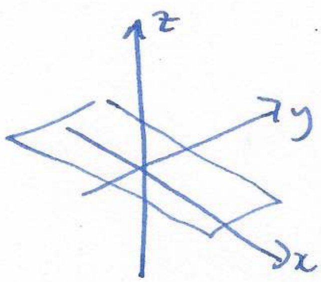
line: $\underline{r}(t) = \langle 1, 0, 0 \rangle + t \langle 5, 7, 4 \rangle$

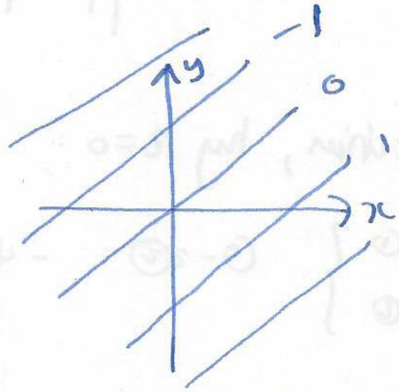
(3) (10 points) Sketch the graphs of the following functions, and their contour maps/level sets.

(a) $f(x, y) = x - y$

(b) $f(x, y) = -x^2 - 4y^2$

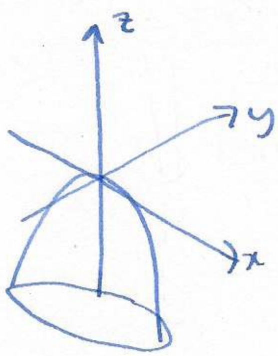
a) $z = x - y$ plane

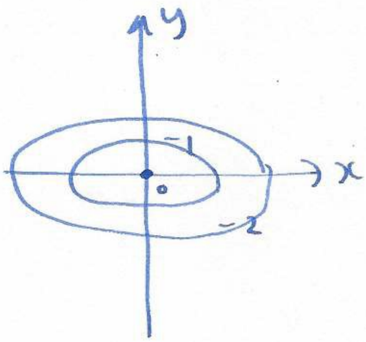




level sets $x - y = c$ $\quad y = x - c$

b)





level sets $-x^2 - 4y^2 = c$

(4) (10 points) Write down a parameterization for the straight line segment from $(3,1,2)$ to $(1,5,6)$. Use the integral formula for arc length to find the length of this line.

$$\underline{r}(t) = \langle 3,1,2\rangle + t\langle -2,4,4\rangle \qquad 0 \leq t \leq 1$$

$$\underline{r}'(t) = \langle -2,4,4\rangle$$

$$\|\underline{r}'(t)\| = \sqrt{4+16+16} = \sqrt{36} = 6$$

$$\text{length} = \int_0^1 \|\underline{r}'(t)\|\,dt = \int_0^1 6\,dt = \left[6t\right]_0^1 = 6$$

(5) (10 points) An object is thrown from the origin with initial velocity $\langle -10, 10, 20\rangle$ m/s. Find an expression for the position of the object at time t it moves under constant gravitational acceleration $\langle 0, 0, -g\rangle$ m/s^2. Feel free to take $g = 10$.

$\underline{x}''(t) = \langle 0, 0, -10\rangle$

$\underline{x}'(t) = \langle 0, 0, -10t\rangle + \underline{c_1}$ $\qquad \underline{x}'(0) = \langle -10, 10, 20\rangle$

$\Rightarrow \underline{c_1} = \langle -10, 10, 20\rangle$

$\underline{x}'(t) = \langle 0, 0, -10t\rangle + \langle -10, 10, 20\rangle$

$\underline{x}(t) = \langle 0, 0, -5t^2\rangle + \langle -10, 10, 20\rangle t + \underline{c_2}$ $\qquad \underline{x}(0) = \underline{0}$

$\Rightarrow \underline{c_2} = \underline{0}$

position: $\underline{x}(t) = \langle -10t, 10t, 20t - 5t^2\rangle$.

(6) Show that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^2}$$

$$\lim_{(x,0)\to(0,0)} \frac{0}{x^2 \neq 0} = 0$$

$$\lim_{(t,t)\to(0,0)} \frac{t^2}{t^2+t^2} = \frac{1}{2}$$

$0 \neq \frac{1}{2} \Rightarrow$ Limit DNE

(7) Find all first partial derivatives for $f(x, y, z) = \sqrt{2xz - 5yz}$. $= (2xz - 5yz)^{1/2}$

$$\frac{\partial f}{\partial x} = \frac{1}{2}(2xz - 5yz)^{-1/2} \cdot (2z)$$

$$\frac{\partial f}{\partial y} = \frac{1}{2}(2xz - 5yz)^{-1/2} \cdot (-5z)$$

$$\frac{\partial f}{\partial z} = \frac{1}{2}(2xz - 5yz)^{-1/2} \cdot (2x - 5y)$$

(8) Find f_{xz} for $f(x,y,z) = \tan^{-1}(3xyz)$.

$$\frac{\partial f}{\partial x} = \frac{1}{1+(3xyz)^2} \cdot (3xyz) = \frac{3yz}{1+9x^2y^2z^2}$$

$$\frac{\partial^2 f}{\partial z \partial x} = \frac{(1+9x^2y^2z^2)(3y) - 3yz(18x^2y^2z)}{(1+9x^2y^2z^2)^2}$$

(9) Find the linear approximation to $f(x, y) = e^{2x+y} + y\sin(4xy)$ at the point $(1, 2)$.

$$L(x,y) = f(1,2) + \frac{\partial f}{\partial x}(1,2)(x-1) + \frac{\partial f}{\partial y}(1,2)(y-2)$$

$$\frac{\partial f}{\partial x} = e^{2x+y} \cdot 2 + y\sin(4xy) \cdot 4y \qquad \frac{\partial f}{\partial x}(1,2) = 2e^{4} + 16\sin 8$$

$$\frac{\partial f}{\partial y} = e^{2x+y} + \sin(4xy) + y\cos(4xy) \cdot 4x \qquad \frac{\partial f}{\partial y}(1,2) = e^{4} + \sin(8) + 8\underset{\cos}{\cancel{\sin}}\, 8.$$

$$L(x,y) = e^{4} + 2\sin(8) + \left(2e^{4} + 16\sin 8\right)(x-1)$$
$$+ \left(e^{4} + \sin 8 + 8\overset{\cos}{\cancel{\sin}}\, 8\right)(y-2)$$

(10) Find the normal vector to the surface $z^2 = 2x^2 + y^2$ at the point $(2, 1, 3)$.

consider $g(x,y,z) = 2x^2+y^2-z^2$

$\nabla g = \langle 4x, 2y, -2z \rangle$

$\nabla g\,(2,1,3) = \langle 8, 2, -6 \rangle$