Math 233 Calculus 3 Fall 15 Midterm 1a

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator without symbolic algebra capabilities, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

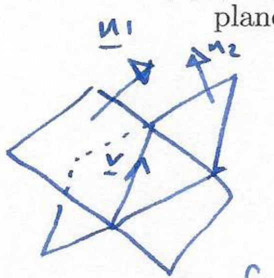
(1) (10 points) Find the angle between the two vectors $\langle 3, -2, -1\rangle$ and $\langle 4, -1, -3\rangle$.

$$\underline{u} \cdot \underline{v} = \|\underline{u}\| \, \|\underline{v}\| \cos\theta$$

$$12 + 2 + 3 = \sqrt{9+4+1}\,\sqrt{16+1+9}\,\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{17}{\sqrt{14}\sqrt{26}}\right) \approx 0.471$$

(2) (10 points) Find a parametric equation for the line of intersection of the two planes $2x - y + 3z = 1$ and $-x + 2y + 2z = -2$.



$$\underline{v} = \underline{n}_1 \times \underline{n}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & -1 & 3 \\ -1 & 2 & 2 \end{vmatrix} = \langle -8, -7, \cancel{5}\,3 \rangle$$

find point in intersection

try $z=0$: ① $2x - y = 1$, ② $-x + 2y = 2$

①+2②: $3y = 5$ $\quad y = \frac{5}{3}$.

$x = \dfrac{1 + \frac{5}{3}}{2} = \frac{4}{3}$.

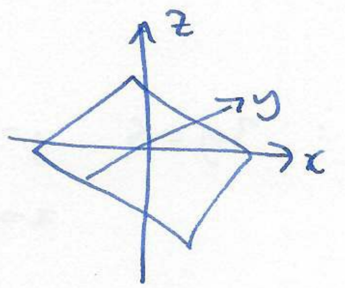
line: $\underline{r}(t) = \langle \frac{4}{3}, \frac{5}{3}, 0 \rangle + t \langle -8, -7, \cancel{8}\,3 \rangle$

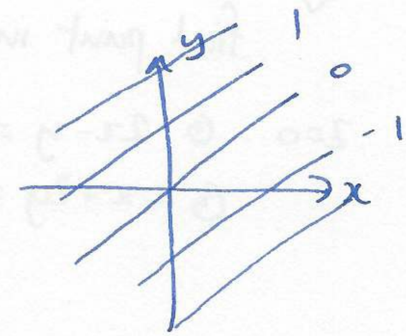
(3) (10 points) Sketch the graphs of the following functions, and their contour maps/level sets.

(a) $f(x,y) = y - x$

(b) $f(x,y) = -4x^2 - y^2$

a) $z = y - x$ plane

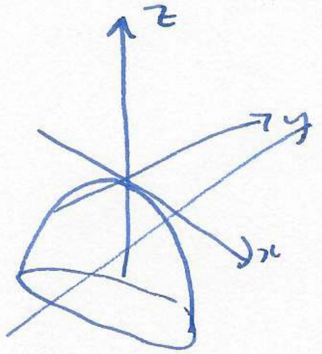


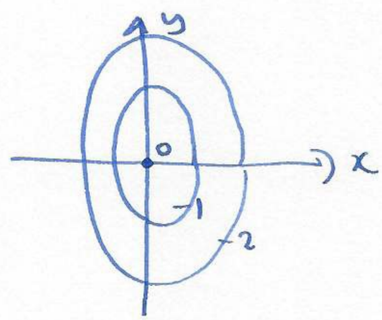


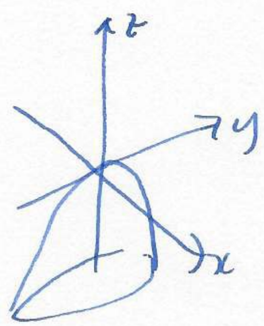
level sets $y - x = c$

$y = x + c$

b)







↑ more accurate.

level sets $-4x^2 - y^2 = c$

(4) (10 points) Write down a parameterization for the straight line segment from (3, 4, 2) to (2, 5, 8). Use the integral formula for arc length to find the length of this line.

$$\underline{r}(t) = (1-t)\langle 3,4,2\rangle + t\langle 2,5,8\rangle$$

$$= \langle 3,4,2\rangle + t\langle -1,1,6\rangle \quad 0 \leq t \leq 1$$

$$\underline{r}'(t) = \langle -1,1,6\rangle$$

$$\|r'(t)\| = \sqrt{1+1+6^2} = \sqrt{38}$$

$$\text{length} = \int_0^1 \|r'(t)\|\,dt = \int_0^1 \sqrt{38}\,dt = \left[\sqrt{38}\,t\right]_0^1 = \sqrt{38}$$

(5) (10 points) An object is thrown from the origin with initial velocity $\langle 10, -10, 20\rangle$ m/s. Find an expression for the position of the object at time t it moves under constant gravitational acceleration $\langle 0, 0, -g\rangle$ m/s^2. Feel free to take $g = 10$.

$$\underline{x}''(t) = \langle 0, 0, -\overset{10}{g} \rangle$$

$$\underline{x}'(t) = \langle 0, 0, -\underset{10}{g}t \rangle + \underline{c_1} \qquad \underline{x}'(0) = \langle 10, -10, 20 \rangle .$$

$$\Rightarrow \quad \underline{c_1} = \langle 10, -10, 20 \rangle .$$

$$\underline{x}'(t) = \langle 0, 0, -10t \rangle + \langle 10, -10, 20 \rangle$$

$$\underline{x}(t) = \langle 0, 0, -5t^2 \rangle + \langle 10, -10, 20 \rangle t + \underline{c_2} \qquad \underline{x}(0) = \underline{0}$$

$$\Rightarrow \underline{c_2} = 0$$

position: $\underline{x}(t) = \langle 10t, -10t, 20t - 5t^2 \rangle$

(6) Show that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2+y^2}$$

$$\lim_{(x,0)\to(0,0)} \frac{0}{x^2+0} = 0$$

$$\lim_{(t,t)\to(0,0)} \frac{t^2}{t^2+t^2} = \frac{1}{2}$$

$0 \neq \frac{1}{2}$ limit does not exist

(7) Find all first partial derivatives for $f(x, y, z) = \sqrt{3xy - 7yz}$. $= (3xy - 7yz)^{1/2}$

$$\frac{\partial f}{\partial x} = \frac{1}{2}(3xy - 7yz)^{-1/2} \cdot (3y)$$

$$\frac{\partial f}{\partial y} = \frac{1}{2}(3xy - 7yz)^{-1/2} \cdot (3x - 7z)$$

$$\frac{\partial f}{\partial z} = \frac{1}{2}(3xy - 7yz)^{-1/2} \cdot (-7y)$$

(8) Find f_{xz} for $f(x, y, z) = \tan^{-1}(2xyz)$.

$$\frac{\partial f}{\partial x} = \frac{1}{1+(2xyz)^2}\cdot(2yz) = \frac{2yz}{1+4x^2y^2z^2}$$

$$\frac{\partial^2 f}{\partial z\partial x} = \frac{(1+4x^2y^2z^2)(2y) - 2yz(8x^2y^2z)}{(1+4x^2y^2z^2)^2}$$

(9) Find the linear approximation to $f(x, y) = e^{x+2y} + y\sin(3xy)$ at the point $(1, 2)$.

$$L(x,y) = f(1,2) + \frac{\partial f}{\partial x}(1,2)\,(x-1) + \frac{\partial f}{\partial y}(1,2)\,(y-2)$$

$$\frac{\partial f}{\partial x} = e^{x+2y} + y\cos(3xy).3y \qquad \frac{\partial f}{\partial x}(1,2) = e^{5} + 12\cos(6)$$

$$\frac{\partial f}{\partial y} = e^{x+2y}.2 + \sin(3xy) + y\cos(3xy).3x \qquad \frac{\partial f}{\partial y}(1,2) = 2e^{5} + \sin(6) + 6\cos(6)$$

$$L(x,y) = e^{5} + 2\sin(6) + \left(e^{5} + 12\cos(6)\right)(x-1)$$
$$+ \left(2e^{5} + \sin 6 + 6\cos 6\right)(y-2)$$

(10) Find the normal vector to the surface $z^2 = x^2 + 2y^2$ at the point $(1, 2, 3)$.

consider $g(x,y,z) = x^2 + 2y^2 - z^2$

$\nabla g = \langle 2x, 4y, -2z \rangle$

$\nabla g(1,2,3) = \langle 2, 8, -6 \rangle$