MTH 233 Sample Midterm 1 Solutions

Q1 a) $\underline{u} = \langle -2,3,4\rangle \qquad \underline{v} = \langle 1,-2,3\rangle$

$$\text{proj}_{\underline{v}}\underline{u} = \frac{\underline{u}\cdot\underline{v}}{\underline{v}\cdot\underline{v}}\,\underline{v} = \frac{-2-6+12}{1+4+9}\langle 1,-2,3\rangle = \frac{4}{\sqrt{14}}\langle 1,-2,3\rangle$$

$$\|\text{proj}_{\underline{v}}\underline{u}\| = \frac{4}{\sqrt{14}}\sqrt{1+4+9} = 4.$$

b) $$\underline{u} = \underbrace{\frac{4}{\sqrt{14}}\langle 1,-2,3\rangle}_{\underline{u}_{\parallel}} + \underbrace{\langle -2,3,4\rangle - \frac{4}{\sqrt{14}}\langle 1,-2,3\rangle}_{\underline{u}_{\perp}}.$$

Q2 a) $\overrightarrow{AB} = \langle 2,4,-4\rangle$

$\overrightarrow{AC} = \langle 4,3,0\rangle$

area $\Delta ABC = \|\overrightarrow{AB}\times\overrightarrow{AC}\|$

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 4 & -4 \\ 4 & 3 & 0 \end{vmatrix} = \langle 12,-16,-10\rangle$$

area $= \sqrt{12^2+16^2+10^2}$.

b) $\underline{n} = \langle 6,-8,-5\rangle$

$\underline{n}\cdot(\underline{x}-\underline{p}) = 0 \qquad \langle 6,-8,-5\rangle\cdot(\langle x,y,z\rangle - \langle -1,-2,2\rangle) = 0$

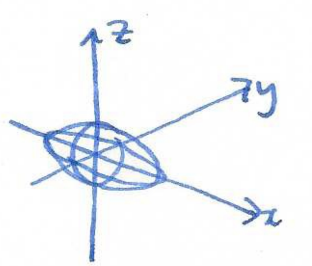
$$6x - 8y - 5z = 0$$

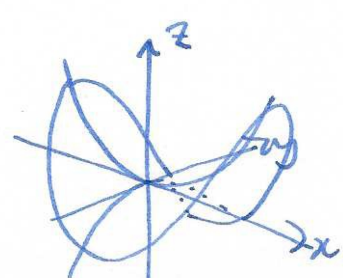
Q3 $\underline{n} = \langle 2,-1,-2\rangle$

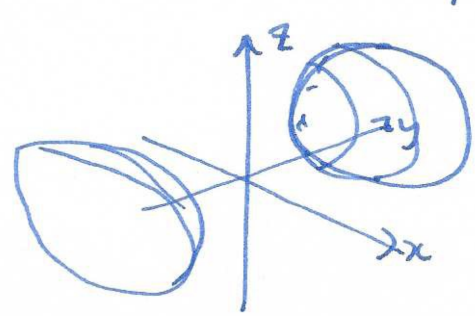
$\underline{n}\cdot(\underline{x}-\underline{p}) = 0 \qquad \langle 2,-1,-2\rangle\cdot(\langle x,y,z\rangle - \langle 5,-7,4\rangle)$

$$= \quad 2x - y - 2z = 9$$

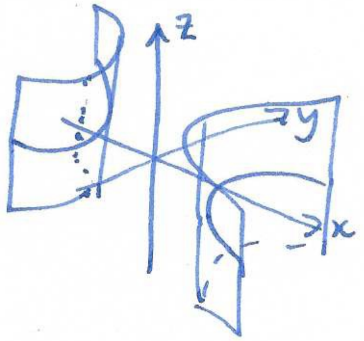
Q4 a) $x^2+4y^2+4z^2 = 16$ b) $z = 9x^2 - 4y^2$ c) $4x^2+9z^2 = 2y^2-72$







d) $9x^2 - 4y^2 = 72$



Q5 $\underline{x}''(t) = 3\underline{i} + 12t^2\underline{j} - 6t\underline{k} = \langle 3, 12t^2, -6t \rangle$

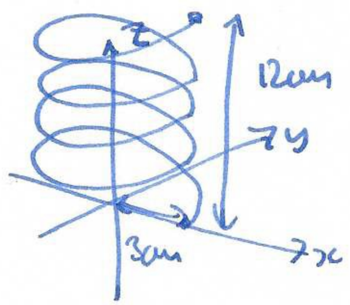
$\underline{x}'(t) = \langle 3t, 4t^3, -3t^2 \rangle + \underline{c} \qquad \underline{x}'(0) = \langle 2, 3, -3 \rangle$

$\underline{x}'(t) = \langle 2+3t, 3+4t^3, -3-3t^2 \rangle$

$\underline{x}(t) = \langle 2t + \frac{3}{2}t^2, 3t + t^4, -3t - t^3 \rangle + \underline{c} \qquad \underline{x}(0) = \langle 1, -2, 3 \rangle$

$\underline{x}(t) = \langle 1 + 2t + \frac{3}{2}t^2, -2 + 3t + t^4, 3 - 3t - t^3 \rangle.$

Q6



a) $\underline{r}(t) = \langle 3\cos t, 3\sin t, \frac{12t}{8\pi} \rangle$

$0 \le t \le 8\pi$

b) $\underline{r}'(t) = \langle -3\sin t, 3\cos t, \underbrace{\frac{12}{8\pi}}_{= \frac{3}{2\pi}} \rangle.$

length $= \int_0^{8\pi} \|r'(t)\| \, dt = \int_0^{8\pi} \sqrt{9\sin^2 t + 9\cos^2 t + \frac{9}{4\pi^2}} \, dt = \int_0^{8\pi} \sqrt{9 + \frac{9}{4\pi^2}} \, dt.$

$= \left[3\sqrt{1 + \frac{1}{4\pi^2}} \, t \right]_0^{8\pi} = 24\pi \sqrt{1 + \frac{1}{4\pi^2}}.$

Q7 $\lim_{(0,y)\to(0,0)} \frac{0}{y^2} = 0 \qquad \lim_{(x,0)\to(0,0)} \frac{x^2}{x^2} = 1 \qquad 0 \ne 1$ ~~at o/s~~ limit DNE.

Q8 $f(x,y,z) = y e^{xy+z} + (x+z)\tan(y+z)$

$f_x = y e^{xy+z} \cdot xy + \tan(y+z)$

$f_y = e^{xy+z} + y e^{xy+z} \cdot x + (x+z)\sec^2(y+z)$

$$f_{xx} = y^3 e^{xy+z} + 0$$

$$f_{xy} = 2y e^{xy+z} \cdot x + y^2 x e^{xy+z} + \sec^2(y+z)$$

$$f_{xyy} = x e^{xy+z} + x e^{xy+z} + x^2 y e^{xy+z} + (x+z) 2\sec(y+z) . \sec(y+z) \tan(y+z)$$

Q9 $\frac{\partial z}{\partial x} = 4x \quad \frac{\partial z}{\partial y} = -8y$

$$z = L(1,1) = f(1,1) + \frac{\partial f}{\partial x}(1,1)(x-1) + \frac{\partial f}{\partial y}(1,1)(y-1) = -2 + 4(x-1) - 8(y-1)$$

Q10 $f(x,y,z) = e^{4yz} + \ln(x+z)$

$$\frac{\partial f}{\partial x} = \frac{1}{x+z}$$

$$\frac{\partial f}{\partial y} = 4z e^{4yz}$$

$$\frac{\partial f}{\partial z} = 4y e^{4yz} + \frac{1}{x+z}$$

$$L(4,-2,1) = f(4,-2,1) + \frac{\partial f}{\partial x}(4,-2,1)(x-4) + \frac{\partial f}{\partial y}(4,-2,1)(y+2) \; \frac{\partial f}{\partial z}(4,-2,1)(z-1)$$

$$L(4,-2,1) = \frac{1}{5}(x-4) + 4e^{-2}(y+2) + \left(-8e^{-2} + \frac{1}{5}\right)(z-1)$$
$$e^{-2} + \ln(5).$$

Q11 $f(x,y,z) = 3x^2 - 2y^2 - z^2 \quad \nabla f = \langle 6x, -4y, -2z \rangle$

$\nabla f(3,-1,5) = \langle 18, 4, -10 \rangle$ normal vector at $(3,-1,5)$.