Math 233 Calculus 3 Fall 2015 Sample Final

(1) Find the line of intersection between the planes $z = 2x-4y+2$ and $x-y-2z = 4$.

(2) Sketch the level sets of the function $f(x, y, z) = x^2 - y^2 - z^2$, and calculate the gradient vector at the point $(2, 1, 1)$. Use this to find the tangent plane to $x^2 = y^2 + z^2 + 2$.

(3) You are driving anticlockwise around a circular roundabout of radius 10m, at 10m/s. When your car is facing due north, you throw a tennis ball from the car due east at 20m/s, at an angle of $\pi/3$ from horizontal. Where does the tennis ball land?

(4) Let $f(x, y) = x^2 + 2y^2 - 2y + 4$.

 (a) Find the critical points of f in the region $x^2 + y^2 < 4$, and use the scond derivative test to classify them.

 (b) Use Lagrange multipliers to find the extreme points on the boundary $x^2 + y^2 = 4$.

 (c) Use your answers above to find the extreme values of f on $x^2 + y^2 \leq 9$.

(5) Change the order of integration to evaluate $\displaystyle\int_0^8 \int_{\sqrt[3]{y}}^2 \cos(x^4)\, dx\, dy$.

(6) Write down triple integrals over the following regions.

 (a) The region inside the sphere $x^2 + y^2 + x^2 = 4$ between the planes the planes $z = 0$ and $z = 1$.

 (b) The volume inside the cylinder $x^2 + y^2 \leq 9$, above $z = 0$ and below $4x + 2y + z = 100$.

 (c) The volume of $z = 8 - 2x^2 - 2y^2$ in the positive octant.

(7) Integrate the vector field $\mathbf{F} = \langle -y, x, z^2 \rangle$ over the paraboloid $z = x^2+y^2$ with $0 \leq z \leq 4$.

(8) Let C be the boundary of the triangle in the plane with vertices $(0, 0)$, $(1, 0)$ and $(1, 3)$. If $\mathbf{F} = \langle \sqrt{1 + x^3}, 2xy \rangle$, use Green's Theorem to evaluate

$$\int_C \mathbf{F}\ d\mathbf{s}.$$

(9) Let $\mathbf{F} = \langle y^2, x, z^2 \rangle$. Let S be the part of the paraboloid $z = x^2 + y^2$, below the plane $z = 1$, with the upward pointing normal. Verify Stokes' Theorem in this case by directly evaluating both integrals.

(10) Let E be the solid cylinder $x^2+y^2 \leq 1$ with $0 \leq z \leq 3$, and let $\mathbf{F} = \langle x, y, -z \rangle$. Verify the divergence theorem by directly evaluating both integrals.