Isolated points

Def$^{\underline{n}}$ $E \subset \mathbb{R}$ $x \in E$ is isolated if there is an interval $(x-c, x+c)$ s.t. $(x-c, x+c) \cap E = \{x\}$ $(c>0)$.

Examples Find isolated points in (a,b), $[a,b]$, $\mathbb{N}$, $\mathbb{R}$, $\mathbb{Q}$., $0 \cup \{\frac{1}{n} \mid n \in \mathbb{N}\}$.

Accumulation points

sometimes every nbd of x contains a point of E. (contains ∞-many?).

Def$^{\underline{n}}$ $E \subset \mathbb{R}$, a point $x \in \mathbb{R}$ (not nec. in E) is an accumulation point of E if every nbd of x contains a point of E.
i.e. for every $c>0$ there is an $t \in E$ s.t $0 < |t-x| < c$.

Examples (a,b), $[a,b]$, $\mathbb{N}$, $\mathbb{R}$, $\mathbb{Q}$, finite set, $\{\frac{1}{n} \mid n \in \mathbb{N}\}$, $(0,1)$, $\{x \in \mathbb{Q} \mid 0 \le x \le \pi\}$

$\{s_n\}$ $s_n = 0$ n odd $s_n = \frac{n}{n+1}$ n even.

Boundary points

Def$^{\underline{n}}$ $E \subset \mathbb{R}$. A point x (not nec in E) is a boundary point of E if every interval $(x-c, x+c)$ contains at least one point in E, and one point not in E.

Examples (a,b), $[a,b]$, $\mathbb{N}$, $\mathbb{R}$, $\mathbb{Q}$.

Example Show that an interior point of E is an accumulation point of E, but not vice versa.

HW: 4.2 1, 2, 4, 5, 8, 10, 13, 20, 21, 24.

§4.3 Sets

Def$^{\underline{n}}$ A set E is closed if it contains all of its accumulation points.

Def$^{\underline{n}}$ $E \subset \mathbb{R}$. Let E' = all accumulation points of E. Then $\overline{E} = E \cup E'$ is called the closure of E

Def$^{\underline{n}}$ A set E is open if every point of E is an interior point of E.

Examples (a,b)