

$\Leftarrow$ Suppose (s_n) is Cauchy.

By HW 2.12.4 (s_n) is bounded.

By Bolzano-Weierstrauss (s_n) has a convergent subsequence (s_{n_k}) , let L be the limit of (s_{n_k}) .

Choose $\epsilon > 0$, and let $N_1 \in \mathbb{N}$ s.t. for all $m, n > N_1$, $|s_n - s_m| < \frac{\epsilon}{2}$.

As $(s_{n_k}) \rightarrow L$, we may choose $N_2 \in \mathbb{N}$ s.t. $|s_{n_k} - L| < \frac{\epsilon}{2}$ for all $k \geq N_2$.

Now choose $N = \max\{N_1, n_{N_2}\}$, and choose $n_k \geq N_2$.

Let $n \geq N_1$, and let $m = n_k$ for some $k \geq N_2$.

Then $|s_n - L| \leq |s_n - s_m + s_m - L| \leq |s_n - s_m| + |s_m - L| = \epsilon \cdot \square$.

HW 2.12 3, 4, 6

§4 Set of real numbers

§4.1 read this.

§4.2 Points

Q: $f(x) = 0 \leftarrow$ solution set is a subset of $\mathbb{R}$, need to describe this.

Defn $[a, b]$ closed interval $\{x \in \mathbb{R} \mid a \leq x \leq b\}$.



(a, b) open interval $\{x \in \mathbb{R} \mid a < x < b\}$



Defn An open nbd (a, b) containing a point x is called a neighbourhood of x .

$(a, b) \setminus \{x\}$ is called the deleted neighbourhood of x .

Note each point $x \in (a, b)$ has many neighbourhoods in (a, b) .

Defn $E \subset \mathbb{R}$. x is an interior point of E if there is a $c > 0$ such that $(x - c, x + c) \subset E$.

Equivalently x is an interior point of E if E contains an (open) nbd of x .

Examples Find interior points in: (a, b) , $[a, b]$, $\mathbb{N}$, $\mathbb{R}$, $\mathbb{Q}$.