§1.8 Induction / §A.8

Let P_n be a sequence of statements depending on $n \in \mathbb{N}$.

<u>step 1</u> check P_1 is true

<u>step 2</u> show that if P_k is true then P_k true $\Rightarrow P_{k+1}$ is true.

then P_n is true for all n.

Examples

① $$1+2+3+\cdots+n = \frac{1}{2}n(n+1) \qquad \sum_{i=1}^{n} i = \frac{1}{2}n(n+1) = f(n).$$

P_1 : $1 = \frac{1}{2}\cdot 1 \cdot 2$ ✓.

P_k : $$1+2+\cdots+k = \frac{1}{2}k(k+1) = f(k)$$

$$1+2+\cdots+k+(k+1) = \frac{1}{2}k(k+1)+(k+1)$$

$$\sum_{i=1}^{k+1} i = (k+1)\left(\frac{1}{2}k+1\right) = \frac{1}{2}(k+1)(k+2) = f(k+2)$$

so $P_k \Rightarrow P_{k+1}$. done ✓.

② show $1^2+2^2+\cdots+n^2 = \frac{1}{6}n(n+1)(2n+1)$.

4. $a_1 = 1$ $\quad a_{n+1} = \frac{1}{3}(a_n + 1)$ show this converges.

HW 2.9. 1 a)–d) typo in a) $-\beta$ should be $-\sqrt{\beta}$.
4.

§2.10 Examples of limits: read this. HW 2.10.1.

§2.11 subsequences

start with a sequence (a_n), get a subsequence by omitting some terms, without changing the order

Example $a_n = \frac{1}{n}$ $\quad \left(\overset{a_1}{1}, \overset{a_2}{\frac{1}{2}}, \overset{a_3}{\frac{1}{3}}, \overset{a_4}{\frac{1}{4}}, \dots\right)$

take every 3rd term $\left(\underset{a_3}{\frac{1}{3}}, \underset{a_6}{\frac{1}{6}}, \underset{a_9}{\frac{1}{9}}, \dots\right)$

Defn Let (s_n) be a sequence. A subsequence of (s_n) is a sequence $(s_{n_1}, s_{n_2}, s_{n_3}, \dots) = (s_{n_k})$ where $n_1 < n_2 < n_3 < \dots$ is an increasing sequence of natural numbers. [note: $n_k \geq k$ for all k.]

Thm A sequence converges to L iff ~~and only~~ every subsequence converges to L.

Q: do you need "converges to L" here?
: how many subsequences?

Proof $\Leftarrow$ (s_n) is a subsequence of (s_n), so (s_n) converges to L.

$\Rightarrow$ suppose (s_n) converges to L, and let (s_{n_k}) be a subsequence.

for any $\epsilon > 0$ there is an N s.t. $|s_n - L| < \epsilon$ for all $n \geq N$.

Same N works for the subsequence ($n_k \geq k$!) so $|s_{n_k} - L| < \epsilon$ for all $k \geq N$. □