<u>Thm</u> (Reciprocal Th^m) Consider a sequence (a_n) of <u>positive</u> numbers $a_n > 0$ for all n. Then $(a_n)_{n\in\mathbb{N}}$ diverges to $+\infty$ iff $\frac{1}{a_n}$ converges to 0.

<u>Proof</u> (HW 2.5.7) (sketch)

$\Rightarrow$ $a_n \to +\infty$ as $n \to \infty$ means

for all M there is an N s.t. $a_n \geq M$ for all $n \geq N$.

~~pick~~ want: for all $\epsilon > 0$ there is an N s.t. for all $n \geq N$ $\frac{1}{a_n} \leq \epsilon$.

choose $M = \frac{1}{\epsilon}$. $a_n \geq M \Leftrightarrow \frac{1}{a_n} \leq \frac{1}{M} = \epsilon$. $\square$.

<u>HW</u> 1. if $r > 1$ show $r^n \to +\infty$
2. if $0 < r < 1$ then $r^n \to 0$

2.5. 3, 7, 8.

§2.6 Boundedness and limits

<u>Def^n</u> A sequence $\{s_n\}$ is <u>bounded</u> if there is a positive real number M such that $|s_n| \leq M$ for all $n \in \mathbb{N}$.

<u>Remark</u> $\Leftrightarrow$ $s : \mathbb{N} \to \mathbb{R}$ bounded as a function.

<u>Def^n</u> A ~~function~~ sequence is <u>unbounded</u> if it is not bounded.

A sequence is <u>bounded above</u> if there is a real number M s.t. $s_n \leq M$ for all n

A sequence is <u>bounded below</u> " " m s.t. $m \leq s_n$ for all n.

<u>Q</u>: $\{s_n\} \to \infty$ vs. s_n unbounded, are these the same?

is there a sequence s_n bounded below, unbounded above, $s_n \not\to \infty$?

<u>Q</u>. does every bounded sequence converge?

<u>Q</u>: can an unbounded sequence converge?, i.e.

<u>Q</u>: is every convergent sequence bounded?

Thm (Boundedness Thm) Every convergent sequence is bounded

Proof Suppose $s_n \to L$ as $n \to \infty$. Choose $\epsilon = 1$, then there is an N s.t. $|s_n - L| \le 1$ for all $n \ge N$

so $|s_n| = |s_n - L + L| \le |s_n - L| + |L| \le 1 + |L|$ for all $n \ge N$.

set $M = \max\{|s_1|, |s_2|, \dots, |s_{N-1}|, 1+|L|\}$. $\square$.

Q: True or false: $s_n > 0$ for all $n \Rightarrow$ there is a c s.t. $s_n > c > 0$ for all n?

Thm (Non-zero limit Thm) Suppose $s_n \to L \neq 0$. Then there is an N s.t. $s_n \neq 0$ for all $n \ge N$. (in fact can find N s.t. $|s_n| \ge \frac{1}{2}|L|$ for all $n \ge N$.

Proof: HW 2.6.5.

choose $\epsilon = \frac{|L|}{2}$, let N be such that $|s_n - L| \le \frac{|L|}{2}$ for all $n \ge N$.

$|s_n| = |s_n - L + L| = |L - (L - s_n)| \ge |L| - |L - s_n| \ge |L| - \frac{|L|}{2} \ge \frac{|L|}{2} > 0$. $\square$.

HW: 2.6.1, 2.6.2, 2.6.5.

Q: If $s_n \to L < 1$ then for every $r \in \mathbb{R}$ $L < r < 1$, $s_n < r$ eventually (in particular $s_n < \frac{1+L}{2}$ eventually).