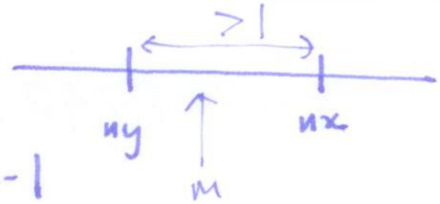
i.e. $n(x-y) > 1$



$nx > ny + 1$ or $ny < nx - 1$

let m be the largest integer less than nx, i.e.

$$nx - 1 \le m < nx$$

so

$$ny < nx - 1 \le m < nx$$

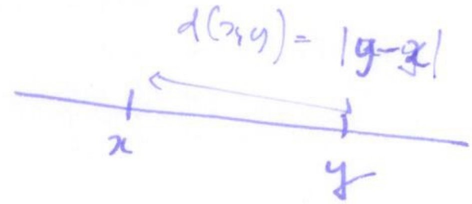
$$y < \underbrace{\frac{m}{n}}_{} < x$$

rational number in (x,y) $\square$.

HW 1.9.1
1.9.5.

§1.10 Metric structure of $\mathbb{R}$

a metric is a distance function



examples

Defn For any real number x,

$$|x| = x \text{ if } x \geq 0$$
$$|x| = -x \text{ if } x < 0$$

Warning: can't just drop negative sign! Ex $|-x| \neq x$ in general!.

$x = -1$: $|-(-1)| \neq -1$.

Useful properties of absolute value:

Thm 1.17

1. For any $x \in \mathbb{R}$, $-|x| \le x \le |x|$
2. For any $x, y \in \mathbb{R}$, $|xy| = |x||y|$.

3. For any $x, y \in \mathbb{R}$, $\quad |x+y| \leq |x|+|y| \quad$ (triangle inequality)

4. For any $x, y \in \mathbb{R}$, $\quad |x|-|y| \leq |x-y|$

and $\quad |y|-|x| \leq |x-y|$.

Remark: $4 \Leftrightarrow \big|\, |x|-|y| \,\big| \leq |x-y|$.

Proofs: can just check all cases $\square$.

Def$^{\underline{n}}$ The distance between two real numbers x and y is

$$d(x,y) = |x-y|$$

Properties

1. $d(x,y) \geq 0 \quad$ (all distances are positive or zero)
2. $d(x,y) = 0$ iff $x=y \quad$ (different points are a positive distance apart
3. $d(x,y) = d(y,x) \quad$ (symmetric)
4. $d(x,y) + d(y,z) \leq d(x,z) \quad$ (triangle inequality)

---

in terms of ~~absolute value~~ norm: ~~let~~

$|a| \geq 0$

$|a| = 0$ iff $a=0$

$|a| = |-a|$

$|a+b| \leq |a|+|b|$.

Practice problems

1. If $a<b$ then $\quad a < \frac{a+b}{2} < b$

2. if $a,b>0$ then $a<b \iff a^2<b^2 \iff \sqrt{a}<\sqrt{b}$

3. if $a,b\geq 0$ then $\sqrt{a^2+b^2} \leq a+b$

4. $|a|\leq b$ iff $-b\leq a\leq b$

5. $|a|\geq b$ iff $a\leq -b$ or $a\geq b$.

HW. 1.10.3,5,7

Read 2.1, 2.2, 2.4.

HW show $a=b$ iff $|a-b|<\epsilon$ for all $\epsilon>0$.

§2 Sequences

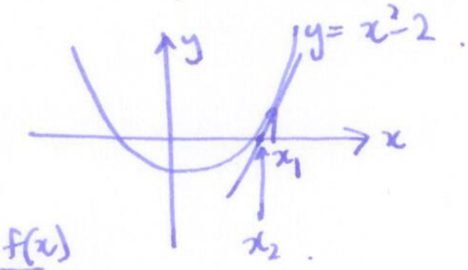
§2.1 Introduction

"everyone can solve an ~~eatha~~ equation in one variable". (false) (e.g. $e^x=\frac{1}{x}$, $x^2=2$?).

however, we can try and find an approximate solution,

e.g. by Newton's method.

~~Example solve $x^2-2=0$~~



given $f(x)=0$, set $F(x)=x-\frac{f(x)}{f'(x)}$

start with guess x_1, compute $x_2=F(x_1)$, $x_3=F(x_2)$, ... etc. hope these get close to a solution.

Example find $\sqrt{2}$, i.e. solve $f(x)=x^2-2=0$ &

so $F(x)=x-\frac{f(x)}{f'(x)}=x-\frac{x^2-2}{2x}$ (goes back 3500 years for $\sqrt{2}$).

gives:

$x_1 = 1.0000000\ldots$ $\qquad x_5 = 1.4142136\ldots$

$x_2 = 1.5000000\ldots$ $\qquad x_6 = 1.4142136\ldots$

$x_3 = 1.4166666\ldots$

$x_4 = 1.4142156\ldots$