

§7.5 More trig equations

Example

$$1 + \sin \theta = 2 \cos^2 \theta \quad \text{use: } \sin^2 \theta + \cos^2 \theta = 1.$$

$$1 + \sin \theta = 2(1 - \sin^2 \theta)$$

$$2 \sin^2 \theta + \sin \theta - 1 = 0$$

$$(2 \sin \theta - 1)(\sin \theta + 1) = 0$$

$$\sin \theta = \frac{1}{2} \quad \sin \theta = -1.$$

Example

$$\sin 2\theta - \cos \theta = 0$$

$$\text{use: } \sin 2\theta = 2 \sin \theta \cos \theta$$

$$2 \sin \theta \cos \theta - \cos \theta = 0$$

$$\cos \theta (2 \sin \theta - 1) = 0$$

$$\cos \theta = 0 \quad \sin \theta = \frac{1}{2}.$$

Example

$$\cos \theta + 1 = \sin \theta \quad \text{try: square}$$

$$(\cos \theta + 1)^2 = \sin^2 \theta$$

$$\cos^2 \theta + 2 \cos \theta + 1 = \sin^2 \theta \quad \text{use } \sin^2 \theta + \cos^2 \theta = 1$$

$$\cos^2 \theta + 2 \cos \theta + 1 = 1 - \cos^2 \theta$$

$$2 \cos^2 \theta + 2 \cos \theta = 0$$

$$2 \cos \theta (\cos \theta + 1) = 0$$

$$\cos \theta = 0 \quad \cos \theta = -1$$

check solns actually work!

$$(\text{note: } x = 1$$

$$x^2 = 1 \Rightarrow x = \pm 1) \quad (\text{but only one works})$$

Example

$$\sqrt{3} \tan \frac{\theta}{2} - 1 = 0$$

$$\tan \frac{\theta}{2} = \frac{1}{\sqrt{3}} \quad \text{ok.}$$