§7.2 Addition and subtraction formulae

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B).$$
$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

Recall $\sin(-A) = -\sin(A)$ $\quad$ $\cos(-A) = \cos(A)$

so
$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$
$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

(prosthaphaeresis formulae)

Examples

$$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right).$$

$$\sin(A+B) + \sin(A-B) = 2\sin A\cos B.$$

~~$\sin(A+B) - \sin(A-B) = 2\cos A\sin B$~~

$$\cos(A+B) + \cos(A-B) = 2\cos A\cos B$$
$$\cos(A+B) - \cos(A-B) = -2\sin A\sin B.$$

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A\cos B + \sin B\cos A}{\cos A\cos B - \sin A\sin B} \quad / \cos A\cos B$$
$$/ \cos A\cos B.$$

$$= \frac{\frac{\sin A\cos B}{\cos A\cos B} + \frac{\sin B\cos A}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}} = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

Example write $-\sqrt{3}\sin x + \cos x$ in terms of $\sin x$ only

$$a\sin(x+b) = a\sin(x)\cos(b) + a\cos x\sin(b)$$

$$\left.\begin{aligned} a\cos(b) &= -\sqrt{3} \\ a\sin(b) &= 1 \end{aligned}\right\} \quad \tan(b) = -\frac{1}{\sqrt{3}} \ \ldots$$

Double angle

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Half angle

$$\tan\left(\tfrac{1}{2}\theta\right) = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

$$\sin\left(\tfrac{1}{2}\theta\right) = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\left(\tfrac{1}{2}\theta\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}.$$

Examples

simplify $\dfrac{2\tan 2\theta}{1-\tan^2 2\theta} = \tan 4\theta$.