

### §7.1 Trig identities

$$\tan x = \frac{\sin x}{\cos x} \quad \sec x = \frac{1}{\cos x} \quad \csc x = \frac{1}{\sin x} \quad \cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

$$\sin^2 x + \cos^2 x = 1 \quad \text{if } \tan^2 x + 1 = \sec^2 x \quad 1 + \cot^2 x = \csc^2 x.$$

#### Examples

1.  $\cos t \tan t = \cos t \cdot \frac{\sin t}{\cos t} = \sin t$

2. write  $\tan^2 x - \sec^2 x$  in terms of sine and cosine:

$$\frac{\sin^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} = \frac{\sin^2 x - 1}{\cos^2 x} = \frac{-\cos^2 x}{\cos^2 x} = -1$$

(a) sum or difference formula, or the product  
(b) using the Pythagorean identities and know that  $\theta = -\pi/2 + k\pi$  for all integers  $k$

$$\begin{aligned} 3. \frac{\sin \theta - \cos \theta}{\cos \theta} &= \frac{\sin \theta - \frac{1}{\sin \theta}}{\cos \theta} = \frac{\sin^2 \theta - 1}{\sin \theta \cos \theta} = \frac{\cos^2 \theta}{\sin \theta \cos \theta} \\ &= \frac{\cos \theta}{\sin \theta} = \cot \theta \end{aligned}$$

4.  $\frac{\cos y + 1}{1 + \sec y} = \frac{\cos y + 1}{1 + \frac{1}{\cos y}} = \frac{\cos^2 y + \cos y}{\cos y + 1} = \cos y \frac{\cos y + 1}{\cos y + 1} = \cos y$

(a) sum or difference formula,  $\theta = \pi/2 + k\pi$  and  $\theta = \pi/2 + k\pi + \pi/2$

5.  $\frac{\sec x - \cos x}{\tan x} = \frac{\frac{1}{\cos x} - \cos x}{\frac{\sin x}{\cos x}} = \frac{1 - \cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} = \sin x$

6.  $\frac{2 + \cot^2 x}{\csc^2 x} - 1 = \frac{1 + \csc^2 x}{\csc^2 x} - 1 = \frac{1}{\csc^2 x} + 1 - 1 = \sin^2 x + 1 - 1 = \sin^2 x + 1$