§7.1 Trig identities

$\tan x = \frac{\sin x}{\cos x}$ $\quad \sec x = \frac{1}{\cos x}$ $\quad \operatorname{cosec} x = \frac{1}{\sin x}$ $\quad \cot(x) = \frac{1}{\tan(x)} = \frac{\cos x}{\sin x}$

$\sin^2 x + \cos^2 x = 1$ $\quad$ if $\tan^2 x + 1 = \sec^2 x$ $\quad 1 + \cot^2 x = \operatorname{cosec}^2 x$.

Examples

1. $\cos t \tan t = \cos t \cdot \frac{\sin t}{\cos t} = \sin t$

2. write $\tan^2 x - \sec^2 x$ in terms of sine and cosine:

$$\frac{\sin^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} = \frac{\sin^2 x - 1}{\cos^2 x} = \frac{-\cos^2 x}{\cos^2 x} = -1$$

3. $$\frac{\sin\theta - \operatorname{cosec}\theta}{\cos\theta} = \frac{\sin\theta - \frac{1}{\sin\theta}}{\cos\theta} = \frac{\sin^2\theta - 1}{\sin\theta\cos\theta} = \frac{\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\cos\theta}{\sin\theta} = \cot\theta$$

4. $$\frac{\cos y + 1}{1 + \sec y} = \frac{\cos y + 1}{1 + \frac{1}{\cos y}} = \frac{\cos^2 y + \cos y}{\cos y + 1} = \cos y \, \frac{\cos y + 1}{\cos y + 1} = \cos y$$

5. $$\frac{\sec x - \cos x}{\tan x} = \frac{\frac{1}{\cos x} - \cos x}{\frac{\sin x}{\cos x}} = \frac{1 - \cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} = \sin x$$

6. $$\frac{2 + \cot^2 x}{\operatorname{cosec}^2 x} - 1 = \frac{1 + \operatorname{cosec}^2 x}{\operatorname{cosec}^2 x} - 1 = \frac{1}{\operatorname{cosec}^2 x} + 1 - 1 = \sin^2 x$$